

# ‘FAQ’ Modeling of a Melting Snowman\*

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The acronym ‘FAQ’ stands for Fields As Quanta—a new conceptual and computational approach being developed by us *ab initio* [1][2][3][4]. In this paper, we first discuss how the FAQ approach to diffusion fields can be obtained starting from our earlier work concerning probabilistic representation of the Huygens process. The discussion notes the similarities and differences in the evolution of the Dirac pulse in the wave and the diffusion fields. We then computationally model a 3D snowman melting in still air. This seemingly simple problem still involves transient heat conduction, continuously changing size and shape of the domain, and incipient heat absorption. We indicate the relation of FAQ with the existing techniques like cellular automata, FDM, etc. After briefly mentioning some issues including instability, we juxtapose simulated contours with experimental photographs.

## I. THE HELMHOLTZIAN CLASS OF PARTIAL DIFFERENTIAL EQUATIONS

There are deep inter-relations between the wave, diffusion and Poisson-Laplace equations. The inter-relations can be made explicit by looking at their physical meaning, derivations, mathematical forms, or solutions.

Wave Equation [electromagnetic (heat) radiation]

$$\nabla^2 \psi = \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} \quad (1)$$

Diffusion Equation [transient heat conduction]

$$\nabla^2 \theta = \frac{1}{D} \frac{\partial \theta}{\partial t} \quad (2)$$

Poisson-Laplace Equation [steady-state conduction]

$$\nabla^2 \phi = f(x, y, z) \quad (3)$$

Arranged thus, it is easy to see that on the LHS, each equation carries the same term—a Laplacian. On the RHS, once the constants of scaling and are ignored, the systematic decrease in the order of the partial time derivative becomes obvious.

Further, by separation of variables, it can be seen that the space-dependent part for both Eqs. (1) and (2) is governed by the same, Helmholtz equation:

$$\nabla^2 \Omega + \sigma^2 \Omega = 0 \quad (4)$$

where  $\sigma$  is the separation constant.

The Helmholtz equation reduces to the Poisson-Laplace equation if  $\sigma = 0$ .

In view of such deep interrelations, we take Eqs. (1) through (3) to belong together in a single mathematical class. We have named the super-class as Helmholtzian.

For a finite domain, the solution to the Helmholtz equation i.e. Eq. (4) may be represented as a Fourier series:

$$\begin{aligned} \Omega(x) &= \sum_{j=0}^{\infty} \Omega_j(x) \\ &= \sum_{j=0}^{\infty} [C_{xj} \cos(\sigma_j x) + S_{xj} \sin(\sigma_j x)] \end{aligned} \quad (5)$$

The value of  $\sigma$  can be obtained by incorporating boundary conditions. The values of  $C_{sj}$  and  $S_{sj}$  can be obtained using the Euler formulae and the initial value distribution.

In the solutions of Eqs. (1) and (2), the time-dependent terms do differ. For the wave equation, the time-dependent part is:

$$\tilde{T}_j(t) = \pm C_{tj} \cos(\sigma_j ct) \pm S_{tj} \sin(\sigma_j ct) \quad (6)$$

whereas for the diffusion equation, it is:

$$\hat{T}_j(t) = \hat{T}_0 \exp(-\sigma_j \kappa t) \quad (7)$$

Two special solutions to the wave equation are noteworthy. First, for a 3D monochromatic spherical wave, the solution in polar coordinates can be given as:

$$\psi = \frac{\psi_0}{r} \exp[ik(r - ct)] \quad (8)$$

Secondly, d’Alembert’s solution for the Cauchy problem  $\psi(x, 0) = f(x)$  and  $\dot{\psi}(x, 0) = g(x)$  is given by:

$$\psi(x, t) = \frac{1}{2} [f(x - ct) + f(x + ct)] + \frac{1}{2c} \int_{x-ct}^{x+ct} g(\xi) d\xi \quad (9)$$

where  $f(x + ct)$  is the so-called accelerated wave. Eq. (9) can be shown to follow from Eqs (5) and (6).

If the domain is infinite, the Fourier series given in Eq. (5) is to be replaced by the Fourier integral.

According to the hypothesis of separation of variables, all the differences between the wave and diffusion phenomena can be accounted for by the differences in their *time-dependent* parts alone.

## II. EVOLUTION OF THE DIRAC PULSE IN WAVE AND DIFFUSION FIELDS (STANDARD DESCRIPTION)

Consider a 3D point source that may radiate either waves or some diffusing species. Consider the limiting scenario in which the source emits a Dirac delta “function” [5] of finite energy content over an infinitesimally short duration  $dt$ . The Fourier transform of the Dirac delta pulse is a flat spectrum comprised of all frequencies, right “up to” infinity. Depending on whether the domain is finite or infinite, the spectrum may be discrete or continuous.

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Even though the Dirac pulse is infinitely narrow, in the Fourier theory, it is considered to have support over the entire domain.

The evolution of the pulse during a finite time interval  $\Delta t$  is different in the wave and the diffusion fields.

If it is the diffusion field, the solution given in Eqs (5) and (7) does not involve translation with time. Instead, the solution predicts a wavenumber-dependent decay with time. Thus, the sharp spatial features corresponding to lower wavenumbers decay faster. Accordingly, the Dirac pulse gets “degenerated” into a spatially spread out distribution, thus showing that diffusion blunts the initially sharp edges. The solution covers all parts of the domain within infinitesimal time. The distant portions of the domain may have very small values, but these still are finite. With further passage of time, the distribution keeps becoming flatter. However, since there is no translation involved in the solution, the main hump of the distribution remains centered at the same point where the singularity of the Dirac pulse once was.

In contrast, if it is a wave field, none of the infinite frequency components making up the Dirac pulse decays with time. Instead, all the components move simultaneously at the same speed. This, in turn, implies that the time profile of the original signal remains constant, i.e., the pulse shifts without any distortion of shape or blunting.

But in 3D space, an infinitely narrow pulse can only be defined in relation to a surface. For symmetry, assume the surface to be spherical in shape. Suppose that at time  $t = 0$ , the sphere has a radius  $r_1$ . (The sphere can be made as small as desired but it must always exclude the singular source point from the analysis.) After time interval  $\Delta t$ , the sphere grows to the radius  $r_1 + c\Delta t$ .

According to the standard description, the pulse has moved outwards from the source following an abstract light cone in the 3+1D space-time. A 1D space-1D time representation of the cone is shown in Fig. 1. The outward movement of the pulse follows along the angled boundaries.

Note that in the standard description, only the retarded (i.e. the outwardly moving) wave is considered. The accelerated term of d’Alembert’s solution is simply dropped from the discussion because the latter, in the standard description, makes no “physical sense,” e.g. pp. 992 in [6].

Now, by the principle of conservation of energy, as the moving pulse distributes over an ever greater spherical surface, its energy per unit area must decrease. Accordingly, the inverse square law for *energy*, or the  $1/r$  law for wave *amplitudes*, is directly obtained. Mathematically, the source is considered to emit an infinite number of pulses whose respective strengths vary sinusoidally in time. The complete description thus conforms to Eq. (8).

The interesting part to note is that a single Dirac pulse evolves quite differently in wave and diffusion fields even though the space-dependent part for both the equations is governed by the same (Helmholtz) equation.

The inter-relation between the wave and diffusion fields becomes easier to see if, instead of the above description, an alternate process involving a cascade of splitting pulses and their feedbacks, is imagined.

### III. THE HUYGENS PROCESS DESCRIBED USING AN INFINITE TRAIN OF DIRAC’S PULSES

Consider the three Huygens processes—First through Third—as described in [2].

Notably, in all the three processes the Huygens wavelet is isotropic [1], with radius of one wavelength  $\lambda$ . To have space- and time-dependent parts separated already, assume a monochromatic (i.e. sinusoidal) radiation. In this paper, we provide some more details about the First Huygens process including a brief account of the phases.

Refer to Fig. 2 and the accompanying description. The  $x$ -axis can be taken to represent any arbitrary 3D line passing through the origin. According to Fig. 2, as soon as the pulse moves a small distance  $\delta\lambda/2$ , it splits up into accelerated and retarded pulses, each of equal strength. Some part of the split-up portions recombines again after time  $2\delta t$ . As  $\delta t \rightarrow 0$ , the dots in Fig. 2 crowd closer together to fill the entire spatial region corresponding to a single wavelet.

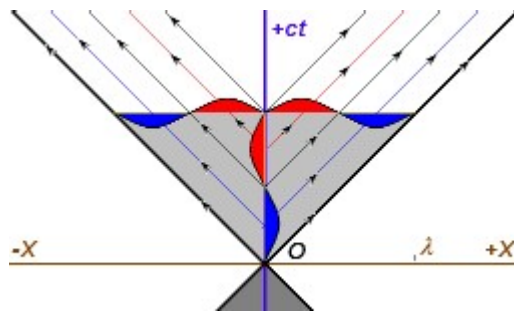


FIG. 1: Space-time diagram of the solution to the wave equation, in the standard view. A single Dirac pulse expands and moves along the  $\pm 45^\circ$  lines. The phase of each Dirac pulse remains constant. The accelerated component of d’Alembert’s solution is simply dropped out of consideration without much ado [6]

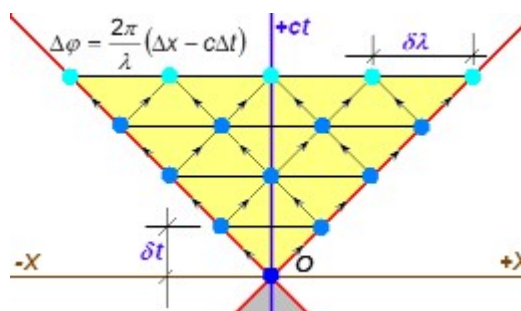


FIG. 2: Our view. Each Dirac pulse continually splits into retarded and accelerated components, which later on recombine at different space-time points to fill the entire yellow region. Both kinds of split-up components continually undergo phase changes:  $\Delta\phi = \frac{2\pi}{\lambda}(\Delta x \pm c\Delta t)$

Now, observe the symmetry about the time or  $ct$  axis in Fig. 2. In a 3D domain, a pair of dots equidistant from the origin implies that they lie on one and the same sphere. Accordingly, the volume of a single Huygens wavelet should be imagined to be composed of an infinite number of concentric spheres. The given pulse would redistribute over all of these spheres over a single wave period  $T$ . As was noted in [1], there are an infinite number of wavelets in a finite region. The *volume* of each

of these wavelets is now being asserted to consist of an infinite number of pulse emitters.

It seems that such a solution should blow up. But in fact it does not because (i) the pulses split up conserving energy and (ii) all the split-up pulses have come from the same original pulse.

In reference to Fig. 2, notice that as the initial pulse is thus redistributing itself over the entire yellow region, the source also remains active along the vertical axis, emitting the subsequent Dirac's pulses. It might be fun to pick an arbitrary dot in Fig. 2 and trace back all the possible original pulses emitted at different times from which the disturbance at the dot is composed.

Since a part of each split-up pulse moves backwards towards the source, the accelerated wave forms an integral part of the above description. But this theoretical integration comes at a cost.

First, in the standard description, the phase alters only when the next Dirac pulse is emitted at the origin. Thus, each Dirac pulse is a phase-wise distinguishable entity, its identity set for all times by the instant of its emission. In contrast, in the present description, the energy content of the pulse seems to vary sinusoidally as the outermost reach of a given Dirac pulse spreads. But actually, such a sinusoidal variation occurs because the original pulse itself no longer exists in its pristine form—it has been splitting up at every instant. The secondary sources are time- or distance-wise separated from their primary sources and therefore differ in instantaneous values of phase. Mathematically, it is not very relevant whether we attribute the phase lag to the separation in space-time of primary-secondary source pairs, or to some hypothetical process actually happening at the secondary source point itself. In either case, the mathematical form of the result is the same: distinct points remain phase-wise different. In short, time and distance effectively alters the phase of the Dirac pulse.

Secondly, unlike the standard description, the instantaneous disturbance at a point cannot be uniquely identified with a single pulse. The instantaneous disturbance at a given point comes about out of all the sub-pulses emitted by all the sources within one  $\delta\lambda$  distance and one  $\delta t$  in the past. Though traceability to a single pulse is lost, the temporal order is not. For example, in Fig. 2, the arrows along the sides of the small triangles all are aligned to the flow of time.

Despite the abovementioned “costs”, we do gain a lot in theory.

First, the support of each pulse can be taken to be *finite* at all times. We thus have a local theory of waves; the FAQ approach completely does away with the premise of instantaneous action-at-a-distance [3].

Note that since FAQ is a local theory, computer simulations using it will in principle show some discrepancy from any analytical solutions that are derived using the Fourier theory.

Now, if the split-up pulse is statistically sampled in space for particles and if these particles are followed up through time, we directly have the resolution of the quantum wave-particle duality of light. Recognizing temporal order leads to a new class of predictions [2].

As another advantage, the wave and the diffusion fields can be related to each other far better.

#### IV. THE HUYGENS PROCESS AND DIFFUSION

To get a diffusion field out of a wave field, one only has to average out the local variations due to phases, that's all!

What kind of emission does that imply *at* the source? The sinusoidal variation may be replaced by, say, a constant RMS value.

The concept of period no longer applies because there are no distinguishing features in the signal itself. However the notion of a characteristic time period and characteristic distance still retain some meaning. They denote the time and space intervals over which the initial flux at the source point characteristically gets distributed. The longer the characteristic distance, the greater the diffusive penetration. However the real importance of retaining the two concepts is that they serve to remind the finite nature of the support [3].

The total flux is, of course, conserved, not only as a time-averaged quantity but also instantaneously.

The rest of the description of the process remains the same as given in [2].

We finally wish to note one more requirement which must be satisfied if the Huygens process is to be used in modeling a classical field. It is that all the Huygens wavelets possible from a given wave-front must radiate simultaneously. Provided this condition is fulfilled, we get a “FAQ” description of classical phenomena—waves or diffusion. Once the simplification of disregarding phases is effected, the First Huygens process directly provides a continuum mechanism of diffusion. By implication, the Second Huygens process describes a probabilistic view of the continuum diffusion. Finally, the Third Huygens process directly describes diffusion of discrete particles under the mathematical Brownian movement (i.e. a Wiener process).

#### V. SO, IS IT JUST RANDOM WALKS AND MONTE CARLO ALL OVER AGAIN?

When it comes to modeling of diffusion fields, it is unusual to find the idea of discrete particles being put forward. Both Rayleigh (1899) and Einstein (1905) had theoretically discussed the idea long before von Neumann, Ulam and Metropolis devised the Monte Carlo algorithm (1946) [7]. In spite of such a long tradition, there are quite a few things which are novel in our research and unique to our approach.

In our theory, we begin by giving a stochastic description of wave fields—not diffusion. We then derive the diffusion field as a simplification of the Huygens process. Both these aspects are novel.

The wave equation involves time to the second order differential whereas the diffusion equation involves it only to the first order. If a statistical view for a higher order field can at all be formulated, then obtaining a similar view for a lower-order field should be easy simply by time-averaging (or “steading”) the higher order field. But the converse would be difficult—one wouldn't know precisely where in theory to introduce the necessary “unsteadiness” or “wiggly-ness”. It is no accident that 46 years after Metropolis *et al*'s work on simulating neutron diffusion, Zwillinger's handbook [8] still listed Monte Carlo

methods only for the parabolic and elliptic equations—not hyperbolic.

Indeed, our approach yields a direct description of waves unlike the other notable particles-based approach of Particles-In-a-Cell (PIC) method.

Overall, the FAQ approach gives a uniform description of all Helmholtzian fields, thereby helping understand even the established analytical and stochastic theories better [3].

Furthermore, FAQ can adapt itself to both classical and quantum mechanical [2] views of waves.

Finally, we explicitize the physical meaning or interpretations of certain fundamental mathematical terms like “Laplacian,” “divergence,” etc. This also makes it easier to extend the FAQ approach to tensor fields—i.e. stress fields [4].

For the same reason, we are able to develop several algorithmic variations to each physical field problem and also provide physical justifications to the programming heuristics such as smoothening.

Finally, by taking care to establish the conceptual relations of FAQ to other techniques such as Cellular Automata (CA), Monte Carlo (MC), FDM, etc., our work helps inter-relate these other existing techniques as well.

## VI. THE SNOWMAN AND THE MELTING EXPERIMENT

The snowman was in the form of a cylinder of 48 mm base diameter, 46 mm height, and a small taper / draft of about 4 mm over the height. The cylindrical form was chosen deliberately. The sharp corners at the top and bottom faces can then be observed for smoothening, something that is not easily possible with ovoid lumps.

Further, our “snowman” was not prepared by compacting snow. Instead, it was obtained by freezing ink water in a plastic mold.

After freezing, the snowman was separated from the mold and kept on a larger cylindrical pedestal of ice made from clear water. (The pedestal dimensions were 75 mm diameter and 45 mm height.) The assembly of the “snowman” and the pedestal was super-cooled to  $-8^{\circ}\text{C}$ , and held at that temperature for 12 hours to remove any internal temperature gradients.

The super-cooled assembly was then brought out of the fridge and kept on a plastic dish. The plastic dish remained inside a closed room. The room received mild diffuse daylight and carried negligible ventilation or air currents. The temperature inside the room remained  $27 \pm 1^{\circ}\text{C}$  throughout the duration of the experiment (about 3 hours).

The assembly was photographed using an ordinary digital camera at the 640 X 480 pixels resolution. All the photographs were digitally resized so as to get an identical size for the plastic dish in each one of them. Given the symmetry in the geometry of the snowman as well as its size and the melting pattern, the photographed contours could then be taken as more or less true orthographic projections.

Fig. 3 shows the photographs after elapse of 2, 20, 60, 100 and 150 minutes respectively. It took 170 minutes for the snowman (but not the pedestal) to melt completely.

## VII. PHYSICAL CONSIDERATIONS IN THE COMPUTER MODEL

The melting of ice is so ordinary circumstance that its computational modeling seems almost trivial. Yet, this problem involves the following complications: (i) a 3D domain; (ii) the continuously changing shape and size of domain—i.e. a moving source [9] and a moving boundary [10] problem; (iii) incipient heat absorption due to the ice  $\rightarrow$  water phase change; (iv) multiple mode heat transfer to the snowman via both radiation and convection (the two being almost equal in magnitude in this case); and (v) transients in heat conduction. Our FAQ-based model includes all of these factors.

However, to maintain the main focus of the physical model on diffusion, we ignore many other considerations such as the convective currents in the air, the thin film of water which formed at the surface of the melting snowman, *etc.*

If the snowman melts in 3 hours, the estimated value of  $h_{ice \rightarrow air}$  is just about  $5W/(m^2K)$ . This leads to the Biot number of just about 0.1. Why then model for transients?

However, note the extra physical constraint: unless the surface layer of the snowman melts, its interior portions won’t. The heat supplied at the boundary cannot diffuse fast enough because the surface temperature itself cannot rise above  $0^{\circ}\text{C}$ . The internal temperature differences are numerically small, being  $< 8^{\circ}\text{C}$ . However, it is the spatially non-uniform distribution which is crucial to determining the shapes assumed by the snowman during its melting. Also see the section on instability in this context.

The pedestal of the snowman was meant to serve as a much larger, even infinite, “ground.” However, as simulations showed, its volume was not sufficient. Thus, heat transfer from the bottom face of the snowman could not be neglected. Accordingly, the pedestal was also included in the model (though not shown in the pictures here).

## VIII. THE RELATION OF ‘FAQ’ TO CELLULAR AUTOMAT AND FDM

The software models the time evolution of the solution at discrete time steps, much like the explicit schemes of FDM [11].

For ease of geometric modeling and rendering, the present software implementation uses voxels [12]. However, the FAQ approach itself is equally compatible with both unstructured meshes and even metaballs. Furthermore, our present implementation of FAQ is flexible enough to accommodate both particles- and continuum-based metaphors.

There is a certain relation of FAQ to the cellular automata (CA). Consider the specific conditions required for such an equivalence. If in FAQ (i) voxels are used for domain representation, (ii) the diffusion process during the finite time step is restricted to the orthogonal discrete points alone, and (iii) a non-stochastic continuum metaphor is employed, then the resulting FAQ model can be directly described as a CA model. (For a relevant implementation of CA, see [13]). The basic reason for the equivalence is that the algorithmic implementation for the vanishing divergence then becomes identical in both

the approaches.

Now, inasmuch such algorithms are equivalent to the iterative methods like Liebmann’s method (i.e. the Gauss-Seidel iteration), it is easy to see the relation that both FAQ and CA have with FDM.

### IX. INSTABILITY IN SOLUTIONS AND OTHER COMPUTATIONAL CONSIDERATIONS

In contrast to the above, if the metaphor of an indivisible particle is used in a stochastic interpretation, then the FAQ approach reduces to the Monte Carlo (MC) technique.

But in a stochastic particle model of the *moving boundary problem*, conditions can easily become conducive to chaos. The reason is the following.

In a particles-based model, the local density at the border voxels determines which voxel would first undergo the solid  $\rightarrow$  liquid phase change. The algorithm then removes the molten voxel out of the solid domain—thereby changing the very domain geometry itself. The last step provides the crucial element of feedback which is necessary for chaos to occur. Now, if the rate of heat input to the border voxels roughly equals the heat of fusion, the feedback dynamics can reinforce the chaotic instability rather than smoothening the solution through diffusive averaging. For example, one of our software implementations showed loss of cylindrical symmetry during shrinking by melting. The resulting shape of the snowman was irregular—cut from one side alone—and not a square/cube as might be anticipated out of grid-aliasing. The seed of the instability lied in the values of the model parameters and truncation due to integer mathematics.

In spite of the relation of FAQ and FDM noted in the above section, the Courant and von Neumann criteria [8] (for wave and diffusion fields respectively) are of limited utility in analyzing the stability of the FAQ models. The reason is that the Huygens wavelet is spherical in shape. It involves the entire spherical surface and not just a few discretely separated points as in the FDM “molecules.” Consequently, it should be possible to achieve solution stability even in the cases that the cited criteria are *not* satisfied.

However, if a spherical wavelet is to be used in digital computations, certain offsetting considerations also assume importance. First, orthogonal discretization can enter in a subtle way. In modeling for fields (as opposed to finding value at a single point), algorithmically, there is no alternative to clubbing together the locations of particles—and this step introduces some discretization even if only in a partial manner. Secondly, it is not always straightforward to find whether a given particle needs to be reflected at curved domain boundaries, and if reflection is required, to find its reflected position accurately. The algorithms to accomplish the two tasks can become compute-intensive except for very simple domain geometries. The available reflection algorithms are suited for detecting collisions of a few large objects, as in games—not millions or even billions of point-particles as in the continuum field simulation. Much work remains to be done in this area. Indeed, the computer model we used for this paper is very similar to CA precisely because the reflection issues are very easy to tackle in such a formulation.

### X. SIMULATION RESULTS AND COMPARISON TO PRIOR WORK

Fig. 4 shows the numerically predicted contours of the vertical mid-section through the 3D snowman.

The simulation qualitatively reproduces the following features: (i) the early melting near sharp domain corners; and (ii) fairly accurate geometric contours, such as the bell-like shape, even the faintly noticeable localized depression in the pedestal near the snowman (not shown here).

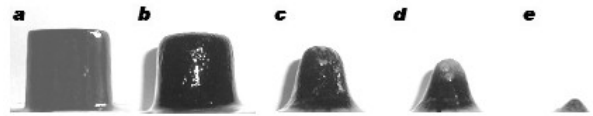


FIG. 3: Photographs of a melting snowman (a) 2, (b) 20, (c) 60, (d) 100 and (e) 150 minutes after taking out of the freeze (i.e.  $-8^{\circ}\text{C}$ ). Complete melting took 170 minutes. The “snowman” was made of ink-water. The clear-ice pedestal on which the snowman rested is not shown here. The surface roughness is due to entrapped air bubbles. Interestingly, due to melting at the ice-air boundary, transients exist despite a low Biot number of just about 0.1.

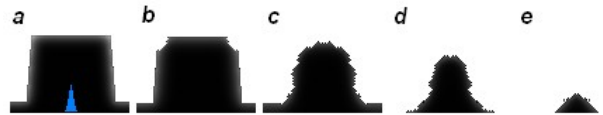


FIG. 4: A coarse-grid computer simulation of the melting snowman. The figure shows vertical mid-sections in a 3D snowman after (a) 76, (b) 116, (c) 202, (d) 288 and (e) 396 steps. Melting began in the 73rd step, and completed in 431 steps. The simulation does not show a time-wise linear conformance to the physical experiment; see the reasons in the main text. The simulation was deliberately kept coarse-grained so as to encourage aliasing during the motion of the melting boundary. Accordingly, the boundary does become jagged. But note that the solution has remained numerically stable, right up to complete melt-down.

However, the simulation shows a somewhat delayed beginning of the melting process. Melting begins as late as in the 73rd step out of the total 431 steps. Yet, in the physical experiment, it is easy to see a thin layer of water form at the surface almost immediately. The discrepancy may be explained in reference to the following: (a) inadequately coarse voxels near the surface; (b) the often different conditions existing at the surface as compared to the bulk of the material, including possibly different values of material parameters like thermal conductivity and heat capacity; and (c) a possibility that a somewhat different form of diffusion equation is applicable at the surface.

As to prior work, Monte Carlo techniques have been used since mid 1960’s. Yet, simulations involving moving boundaries are hard to find. Steady-state moving source problems like zone refining, submerged-arc welding, continuous casting, heating in machining, etc. are often studied. However, problems in which the domain itself undergoes changes are not. The problem of the moving solidification front is a welcome exception, but the differences in the physical contexts makes direct comparisons difficult.

As to melting itself, a recent noteworthy work is [14]. There is a sophisticated melting flow model, but based on a substantially different modeling abstraction. The cited work uses the Marker-and-Cell (MAC) algorithm, a

variation of FDM that is especially suited for free-surface flows. In contrast, here, we are mainly concerned with diffusion in solids, and use a CA-like version of FAQ. The cited model treats both solid and liquid states uniformly via a continuously varying viscosity parameter which becomes infinite for the solid state. In contrast, our model incorporates a sharply demarcated solid  $\rightarrow$  liquid phase change. This difference should make numerical stability somewhat more difficult to achieve in our model because the changes effected to the heat calculations and the domain geometry thereby become rather abrupt in nature. However, it is satisfying to note that despite the ruggedness shown by the moving contours, our model has sufficient robustness that the solution remains stable over the entire time range, right up to complete melting.

## XI. CONCLUDING REMARKS

In this paper, we first presented theoretical details of how phases of waves are accounted for in the FAQ approach. We used a description of the evolution of the Dirac pulse to motivate this discussion. Yet, we did

not therefore repeat the standard Green's function-based description. Our novel assumption that the Huygens wavelet is isotropic simplified the discussion enormously. In particular, we could easily include the accelerated component of d'Alembert's solution. This seems to be a novel development. The discussion then showed how and why the Dirac pulse evolves differently in the wave and the diffusion fields. Thereafter, the stochastic view of diffusion was seen as a simplification of a similar view of waves. This again seems to be a new development. We then computationally modeled a small ( $\sim 5$  cm) snowman melting in still air. The 3D simulation showed several valuable similarities to the physical experiment. We also noted many other aspects, e.g. the relation of the FAQ approach to CA and FDM, the limited applicability of von Neumann's and Courant's criteria, etc. The FAQ model used here is similar to CA. Its formulation was deliberately slanted towards domain aliasing. But despite having moving boundaries, it showed stability over the entire time range. Thus, the FAQ approach was shown to work for a transient and moving boundary problem of heat transfer.

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- [1] Jadhav, A. R. and Kajale, S. R. (2005) "Obliquity factor is not essential to the Huygens-Fresnel principle," *e-Proc. of 50th Congress of ISTAM* held at and pub. by IIT Kharagpur, India
  - [2] Jadhav, A. R. and Kajale, S. R. (2005) "Resolution of the wave-particle paradox of light using a new approach, Parts I and II," *e-Proc. of 50th Congress of ISTAM* held at and pub. by IIT Kharagpur, India
  - [3] Jadhav, A. R. & Kajale, S. R. (2006) "The diffusion equation does not imply instantaneous action at a distance," in preparation, *51st Congress of ISTAM* to be held at AUCE, Visakhapatnam, India
  - [4] Jadhav, A. R. and Kajale, S. R. (2007) "FAQ: A new method of computational mechanics," in preparation, *Intl. Conf. on Advances in Machine Design and Industry Automation*, to be held at COEP, Pune, India.
  - [5] Boas, M. L. (1983) "*Mathematical Methods in the Physical Sciences, 2nd ed.*," John Wiley, New York, USA
  - [6] Feynman, R. P., Leighton, R. B. & Sands, M. (1963) "*The Feynman Lectures on Physics, Vol. II*," Addison-Wesley; reprinted (1986) by Narosa, New Delhi, India
  - [7] Cipra, B. A. (2000) "The best of the 20th century: Editors name top 10 algorithms," *SIAM News*, Vol. 33, No. 4, pp. 1
  - [8] Zwillinger, D. (1992) "*Handbook of Differential Equations, 2nd ed.*," Academic Press, San Diego, CA, USA
  - [9] Nag, P. K. (2002) "*Heat Transfer*," Tata McGraw-Hill, New Delhi, India
  - [10] Greenberg, M. D. (1998) "*Advanced Engineering Mathematics, 2nd Ed.*," Pearson, New Delhi, India
  - [11] Lewis, R. W., Morgan, K., Thomas, H. R. and Seetharamu, K. N. (1996) "*The Finite Element Method in Heat Transfer Analysis*," John Wiley & Sons, Chichester, UK
  - [12] Foley, J. D., van Dam, A., Feiner, S. K. and Hughes, J. F. (1996) "*Computer Graphics: Principles and Practice*," Pearson Education (Singapore), New Delhi, India
  - [13] Weimer, J. R. (2001) "Simulating reaction-diffusion cellular automata with JCASim," in "*Discrete Modelling and Discrete Algorithms in Continuum Mechanics*," Logos-Verlag, Berlin, pp. 217-226
  - [14] Carlson, M., Mucha, P. J., van Horn III, R. B. and Turk, G. (2002) "Melting and flowing," *ACM SIGGRAPH Symposium on Computer Animation* held at San Antonio, TX, USA