# ‘FAQ’ Modeling of a Melting Snowman


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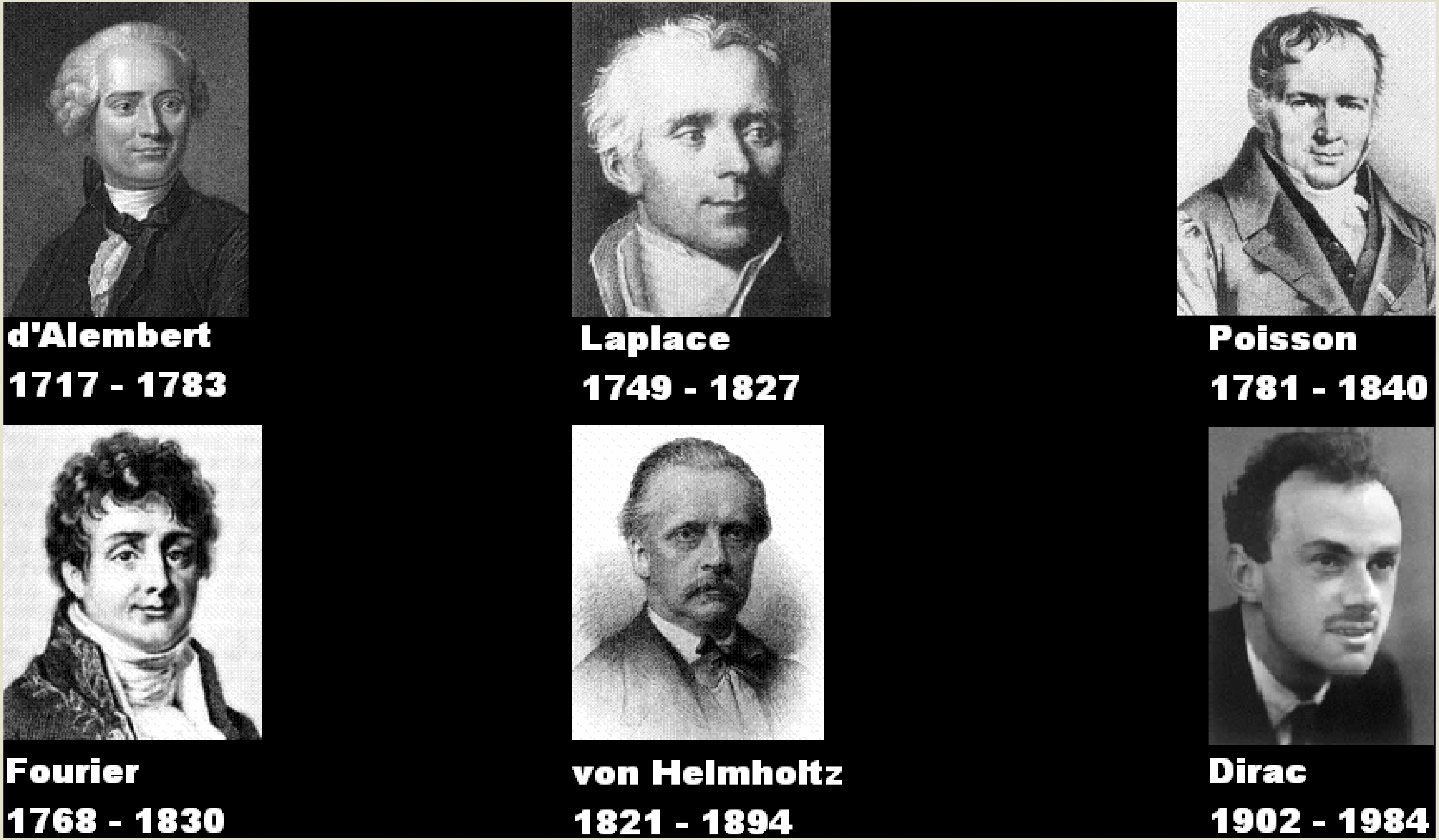
# The Men…


d'Alembert
1717 - 1783
Laplace
1749 - 1827
Poisson
1781 - 1840
Fourier
1768 - 1830
von Helmholtz
1821 - 1894
Dirac
1902 - 1984

# Outline

- What is 'FAQ'?
- Theory
  - The Helmholtzian class of equations
    - Poisson-Laplace, diffusion and wave equations
  - Evolution of the Dirac pulse in waves
  - Description of diffusion from the Huygens process
  - The relation of FAQ to other methods
- Melting Snowman
  - Experiment
  - Computational Modeling
  - Comparison of simulations with experiment
- Conclusions

# 'FAQ' Stands for: Fields As Quanta

- FAQ: A New Method of Computational Mechanics
  - Take any continuum field
  - Replace it by a collection of tiny quanta (and their attributes)

- Why think along such a line?
  - Through sampling, relatively fewer quanta could represent a field
  - Quanta can be simpler units to manipulate computationally
  - Result: Faster, or more efficient, or otherwise better results
  - Quantization is possible for the abstractly defined physical quantities

- Applications
  - Wave fields like light, acoustics, vibrations, etc.
    - Resolution of the quantum wave-particle paradox of light!
  - Potential fields like the ideal fluid flow, electromagnetic fields, etc.
  - Diffusion fields like transient heat conduction, mass diffusion, etc.
  - Tensor fields:
    - A new conjecture
    - Modeling of a plane stress problem

# The Helmholtzian Class: [Wave + Diffusion + Poisson-Laplace]

- This class does not exist!
  - We propose
- The Mathematical Form
  - The LHS terms are similar
  - On RHS, the order of the differential decreases systematically
- The Relations
  - The space-dependent part of both wave and diffusion equations is given by the Helmholtz equation
  - The wave, diffusion and the Helmholtz equation all reduce to the Poisson-Laplace equation

$$\nabla^2 \psi = \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2}$$

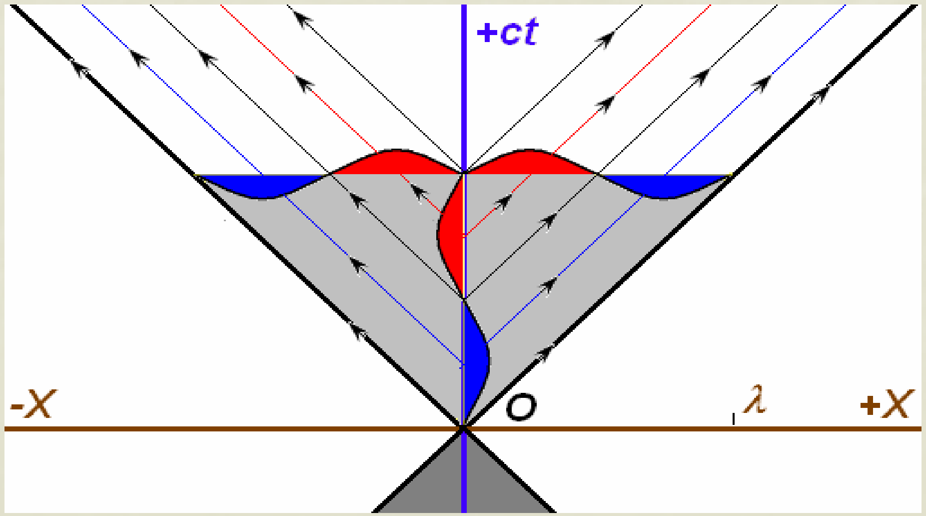
$$\nabla^2 \theta = \frac{1}{D} \frac{\partial \theta}{\partial t}$$

$$\nabla^2 \phi = f(x, y, z) \text{ or } 0$$

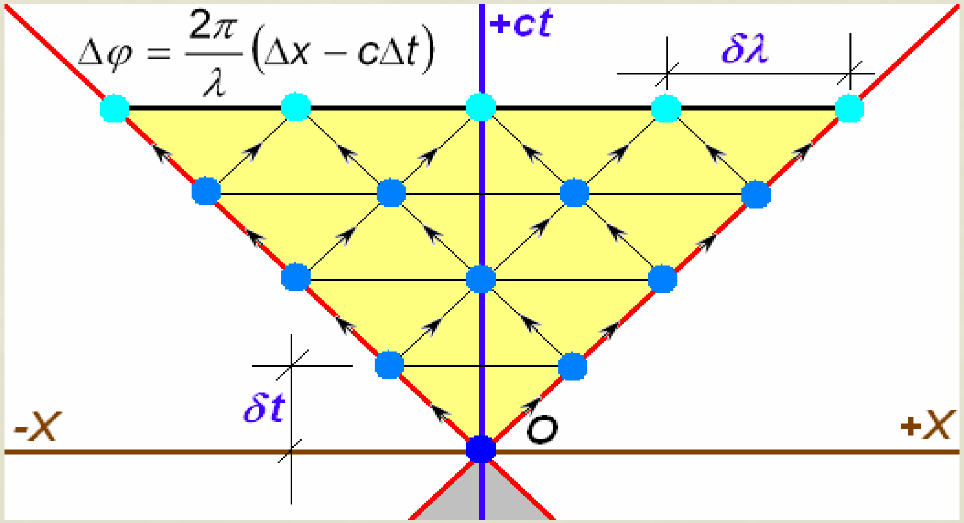
$$\nabla^2 \Omega + \sigma^2 \Omega = 0$$

# Evolution of the Dirac Delta in Waves

- Standard View
  - Each pulse only expands. Maintains the same phase.
  - The accelerated wave is simply dropped out of the consideration




- Our View
  - Each pulse continually splits in both to and fro directions
  - The split up components change phase (with space and time differences)



# Evolution of the Dirac Delta in Diffusion — Standard View

- The Standard View
  - Based on Fourier's Theory
  - The Dirac delta assumed to have support everywhere
  - Hence, the Dirac delta of initial zero width, suddenly spreads all over the domain
  - The higher-frequency spatial harmonics attenuate faster
  - Hence, in place of a point-pulse, we get a hump spread all over the domain
  - The adjustment of the local density with time is taken to be "diffusion"

# Evolution of the Dirac Delta in Diffusion — Our View

- Our View
  - Crucially based on the idea of finitude of support
  - Start with the Huygens process description of waves
  - Take time-average (once) of the field variable
  - Then sample with geometric probability
    - This gives Brownian motion of particles (i.e. MC)
  - The distribution remains finite at all finite times
  - The distribution covers an increasing greater volume of the domain with time—i.e., the support expands.
  - The shape of the hump is necessarily different
  - But, the diffusion equation does get obeyed

# Relation of 'FAQ' to Other Methods - 1

- 'FAQ' is equivalent to Cellular Automata ('CA') if:
  - Voxels are used for domain representation
  - The time variable is discretized uniformly
  - The diffusion process is restricted to orthogonal discrete points
  - A non-stochastic continuum metaphor is employed
- 'FAQ' related to 'FDM'
  - The above version of 'FAQ' and 'CA' are both equivalent to Liebmann's method—i.e. 'FDM'
- 'FAQ' related to Monte Carlo ('MC'):
  - 'FAQ' for diffusion reduces to 'MC' if the metaphor of indestructible discrete particles is employed
  - But there has been no 'MC' equivalent of 'FAQ' for waves

# Relation of 'FAQ' to Other Methods - 2

- 'FAQ' and 'TLM' (Transmission Line Matrix) method
  - In acoustics, 'TLM' is also known as "Discrete Huygens Method"
  - 'FAQ' does not conceptualize continua as networks
    - So, 'FAQ' theory makes no references to lines, nodes & matrices
    - 'FAQ' method does not need to modify the Huygens flows at "nodes"
    - 'FAQ' can easily work with irregular (non-orthogonal) meshes
  - 'FAQ' has many differences of theoretical development
    - The clarification regarding the issue of obliquity factor
    - The resolution of the wave-particle paradox
    - The identification of the issue of compactness of support
    - The extension to tensor fields
  - 'TLM' appears closer to 'CA' than to 'FAQ'

# The Melting of Snowman: Experiment

- “Snowman”: Freeze some ink-water in a cup of
  - ~48 mm Φ, 46 mm height
- “Pedestal”: Freeze some clear water in a bowl
  - ~75 mm Φ, 50 mm height
- Remove both from molds
- Place snowman on its pedestal (as shown)
- Freeze the assembly to -8° C for 12 hours
- Let it melt in the still air inside a closed room at 27 °C. Takes ~3 hrs

# What This "Simple" Situation Involves

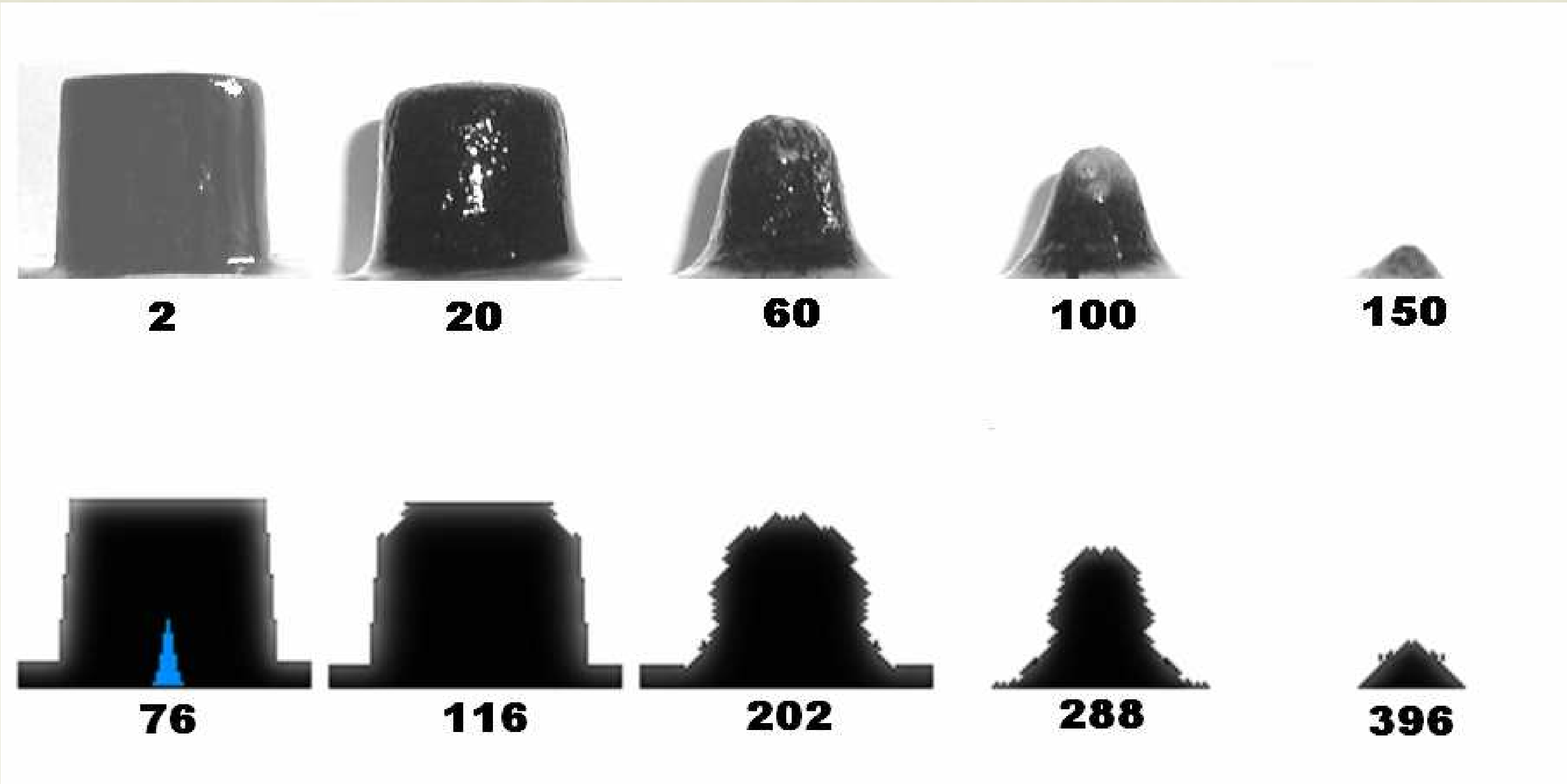
- A 3D domain of irregular shape (not spherical)
- The continuously changing shape and size of domain
  - A moving boundary problem
  - A moving source problem
- Incipient heat absorption
  - The Ice → Water phase change
- Multiple modes of heat transfer
  - Radiation: It is significant here—about 50% of the total flux!
  - Convection: Natural convection. 'h' assumed to be constant
- Transients in conduction
  - Transients exist even if the Biot number is only about 0.1
    - Because, the boundary temperature cannot exceed melting point
  - Else, the snowman won't change shape as it melts!

# Some of the Modeling Considerations

- 3D Model (compute-intensive)
- Voxels for domain representation
  - Tetrahedral or other meshes could have been used
  - Voxels ease the implementation for reflection
- 'FAQ' implementation here happens to be like 'CA'
- Fairly coarse grained mesh
  - Roughly, 1 voxel to 1 cubic mm
  - Coarse granularity should increase the aliasing effects
  - Will this lead to instability? The boundaries are moving!
- We ignored
  - Thin layer of water at the surface
  - Variation of convective coefficient due to natural convection

# Comparison of Simulation with Experiment

- The stages of progress in simulation do not show a linear match with the experiment
  - E.g., melting began relatively late in simulation



# Why the Computational Results Differ

- Too coarse a discretization near the external surface
  - All heat enters from the external surface alone
- In the physical experiment, the conditions at the surface may be different from those in the bulk of the material
  - For example, the local values of thermal conductivity and heat capacity would likely differ from place to place
  - The computational model ignores the thin film of water and its local heating
- A possibility that a somewhat different form of diffusion law may apply for the local heat transfer at the surface
  - The simplest linear equation assumes existence of material on each side. This condition is not met at the surface.
  - May involve additional or more complex flux-gradient relation

# What Does the Model Show Correctly?

- The preferential beginning of melting near sharp corners
- The sequence of different shapes assumed by the snowman during its melting
- The slight depression in the pedestal adjacent to the snowman. (Near the red arrows.)
  - Temperature contours in the computational model show the slight depression. These contours precede the eventual shape.

# What We Discussed…

- The evolution of the Dirac delta in wave fields
  - A new theoretical description
  - The accelerated wave forms an integral part of the description
- A description of the diffusion fields starting from the above
  - The description assumes compact support explicitly
  - The description is not based on the Fourier theory
- ‘FAQ’ simulation and experiment.
  - Simulation shows stability
    - despite having coarse voxels (aliasing) and moving boundaries
  - Simulation shows contours very similar to the experiment
  - But, some mismatch between the number of iterations and time

- ‘FAQ’ modeling works for a fairly complicated problem of transients in heat transfer under moving boundaries

# Thank You!

- Credits
  - The photographs in slide # 2 were taken from the Web pages on history of mathematics maintained by the School of Mathematics and Statistics, University of St. Andrews, Scotland. URL: http://www-groups.dcs.st-and.ac.uk/~history/index.html

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