# Obliquity Factor Is Not Essential to the Huygens-Fresnel Principle*

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We present a new view about the obliquity (or inclination) factor of diffraction theory. The new view identifies obliquity factor with the geometry of the surface of integration, and the values of constraints prescribed on that surface. Thus, the term is taken to indicate certain global aspects of particular field solution(s). As such, the term is neither generally useful in field analysis nor of fundamental relevance in answering whether Huygens' wavelets may be isotropic or not. The reasons follow from a re-examination of how Kirchhoff's theory derives its diffraction formula (which has an obliquity term) starting from the Helmholtz-Kirchhoff Integral Theorem (which does not). We also briefly point out how Huygens' Principle was incorrectly applied in the derivation of Fresnel's theory. Separately, isotropy for Huygens' wavelets is justified on the basis of symmetry in the differential wave equation. Finally, main computational benefits of the present view are briefly indicated.

## I. INTRODUCTION

A new conceptual and numerical approach is *ab initio* being developed for modeling and analysis of engineering field problems on computer [1]. For Helmholtzian (or second-order PDE) fields, the Huygens-Fresnel Principle is at the very base of the formulation of the new approach.

## II. WHY IS OBLIQUITY FACTOR NECESSARY IN THEORY IF IT BREAKS SYMMETRY?

According to standard description in physics texts [2] to [6], obliquity factor (also known as inclination factor) leads to a desirable form of anisotropy in the angular distribution of field strength for the Huygens wavelet, thereby leading to maximum amplitude occurring in the forward direction of the wave travel. The reason given for using the obliquity factor is that doing so ensures that the propagation of wave-front only occurs in the forward direction, not backwards.

But why must wave propagate only in the forward direction? After all, the wave equation carries symmetry: it is symmetrical in a certain sense across both space- and time-dimensions; its spatial differential terms are equal in all space axes. If the governing equation exhibits symmetry, why must its particular solutions not have it? Precisely where does the symmetry get broken?

Here, in physics texts (e.g. see [2]) appeal is made to physical observation. For example, drop a single pebble in a quiescent pond and observe the circular waves that it generates. The waves expand—i.e. they only travel outwards, never inwards; the enclosed circular region remains level. Yet, back-waves, if present, would cause waving within the inner circular region.

In the first ever theory of diffraction, Fresnel (1818) did not derive obliquity factor. Rather, he introduced the idea and assumed it to become zero at angles. The first derivation of the term, for Fresnel's theory, actually came from Stokes (1849) [5]. Later on, Kirchhoff (1882) treated the diffraction problem more rigorously, using the differential wave equation. It was only after Kirchhoff's rigorous theory came forth that the original Huygens' Principle, together with Fresnel's ideas (interference for Huygens secondary waves and obliquity) gained wide acceptance. Today, standard text-book description is to present obliquity factor as a necessary modification to the Huygens-Fresnel wavelets, i.e., as their essential property. Further, perhaps owing to the overall rigor of the Kirchhoff theory, obliquity factor has also been taken to logically arise out of the very nature of wave-fields. For example, see [2] to [5]. Kirchhoff's proof takes a differential field-equation approach, and therefore is of direct relevance in the present research.

## III. KIRCHHOFF'S DIFFRACTION FORMULA: THE GENERIC EXPRESSION FOR OBLIQUITY

We generally follow the treatment and notation in [3, 5]. Consider the scalar wave equation in complex amplitude, $\tilde{\phi}$ , given as: $\nabla^2\tilde{\phi} = \dfrac{1}{c^2}\dfrac{\partial^2\tilde{\phi}}{\partial t^2}$ , where $\tilde{\phi} = \dfrac{\tilde{\phi}_0}{R}e^{ik(R-ct)}$ , $k = \dfrac{2\pi}{\lambda}$, and $R$ is the distance from the source (see Figure 1).

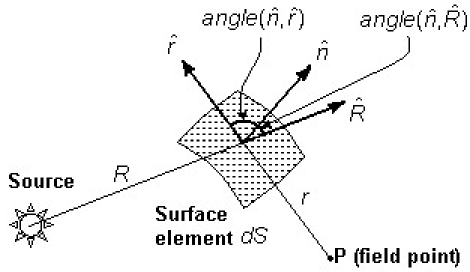


FIG. 1: Geometrical Elements in the Helmholtz-Kirchhoff Integral Theorem

The form of $\tilde{\phi}$ implies that we are considering a cosinusoidal monochromatic radiation, i.e. Fourier theory is assumed to be applicable. The space-dependent part of $\tilde{\phi}$, denoted as $\tilde{U}$, can be isolated as: $\tilde{U} = \tilde{\phi}e^{ikct}$. The spatial distribution of $\tilde{U}$, in turn, is governed by the Helmholtz equation: $\nabla^2\tilde{U} + k^2\tilde{U} = 0$. The Helmholtz equation can be solved using Green's Theorem, to arrive at the Helmholtz-Kirchhoff Integral Theorem:

$$\tilde{U}_P = \frac{1}{4\pi}\left[\oint_S \left(\frac{e^{ikr}}{r}\right)\vec{\nabla}\tilde{U}\cdot d\vec{S} - \oint_S \tilde{U}\vec{\nabla}\left(\frac{e^{ikr}}{r}\right)\cdot d\vec{S}\right]$$

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Here $r$ is the distance of the domain point $P$ from the *surface* element $dS$ (see Figure 1).

Note that the Helmholtz-Kirchhoff Integral Theorem does not carry an obliquity factor term. In fact, discussions of Huygens' Principle in mathematical literature do not even mention the idea; e.g. see [7, 8]. Thus, the present discussion is mainly concerned with physics and its applications.

Substituting the expression for $\tilde{U}$ at point $P$ into Helmholtz-Kirchhoff's Integral Theorem, we get:

$$\tilde{U}_P = \frac{1}{4\pi}\oint_S \left(\frac{e^{ikr}}{r}\right)\frac{\partial}{\partial R}\left(\frac{\tilde{U}_0}{R}e^{ikR}\right)\cos\left(\hat{n},\hat{R}\right)dS$$

$$-\frac{1}{4\pi}\oint_S \left(\frac{\tilde{U}_0}{R}e^{ikR}\right)\frac{\partial}{\partial r}\left(\frac{e^{ikr}}{r}\right)\cos\left(\hat{n},\hat{r}\right)dS$$

Evaluating the differentials under the integral signs, we get:

$$\tilde{U}_P = \frac{1}{4\pi}\oint_S \left(\frac{e^{ikr}}{r}\right)\tilde{U}_0 e^{ikR}\left(\frac{ik}{R}-\frac{1}{R^2}\right)\cos\left(\hat{n},\hat{R}\right)dS$$

$$-\frac{1}{4\pi}\oint_S \left(\frac{\tilde{U}_0}{R}e^{ikR}\right)e^{ikr}\left(\frac{ik}{r}-\frac{1}{r^2}\right)\cos\left(\hat{n},\hat{r}\right)dS$$

Now, if each $R$ and $r$ is far greater than the wavelength $\lambda$, then the terms having $R^2$ and $r^2$ in denominator can be neglected. We thus get the Fresnel-Kirchhoff diffraction formula:

$$\tilde{U}_P = -i\frac{\tilde{U}_0}{\lambda}\oint_S \frac{e^{ik(r+R)}}{rR}\left[\frac{\cos\left(\hat{n},\hat{r}\right)-\cos\left(\hat{n},\hat{R}\right)}{2}\right]dS$$

In the above expression, the square bracket holds the obliquity term in its generic form. Thus,

$$K_\theta = \frac{\cos\left(\hat{n},\hat{r}\right)-\cos\left(\hat{n},\hat{R}\right)}{2}$$

The expression for $K_\theta$ clearly shows that obliquity term refers to the geometrical relation of the given point with the source and the surface of integration. Thus, whenever the specification of the geometry and boundary conditions changes, the relation $K_\theta$ must be separately evaluated. Thus:

*Obliquity factor is a "measure" of the integration scheme being followed for evaluating the surface integrals given by the Helmholtz-Kirchhoff Theorem.*

The evaluation of in the case of an isolated, unobstructed point source in infinite 3D space is relatively easy to carry out.

The surface of integration is taken to consist of two spheres centered on the source (see Figure 2). The inner one is necessary to exclude the singularity at the point source. If the outer sphere is made infinitely large, the surface integral vanishes [5]. Thus, only the inner sphere remains to evaluate the integral on. Observing that the angle is always 180° for a spherical surface of integration, the obliquity factor evaluates to:

$$K_\theta = \frac{\cos\theta + 1}{2}$$

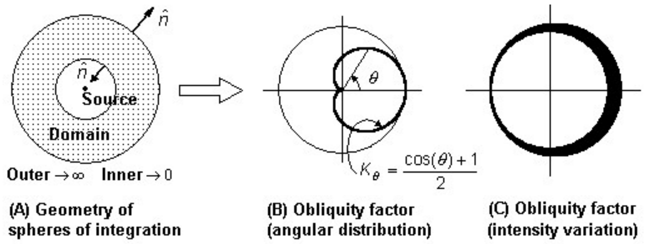


FIG. 2: Obliquity Factor of a Point Source

## IV. KIRCHHOFF'S ANALYSIS OF APERTURE DIFFRACTION

We again follow the explanation given in [5]. Refer to Figure 3, in which an infinite 3D space is divided into two semi-infinite regions, L and R, by an opaque plane carrying a circular aperture.

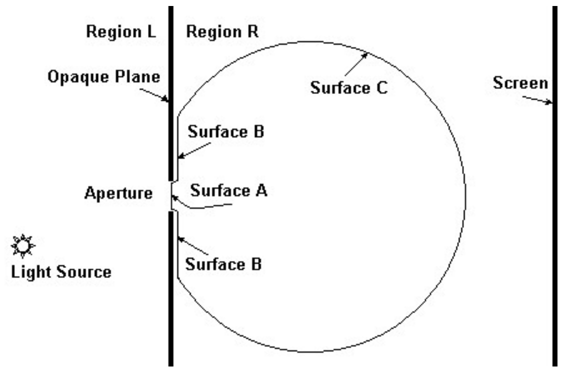


FIG. 3: Kirchhoff's Analysis of Aperture Diffraction (2D Section of Infinite 3D Space)

Light radiation from a source somewhere in the region L impinges on the aperture; the diffracted light propagates into the region R. (Kirchhoff seems to have believed that the aperture-diffracted light propagates only into the region R, not L.) The diffracted light falls on the planar screen S on the right-hand side. Kirchhoff next considers three surfaces joined together for evaluating the integral. The surface A is plane-circular, covering the aperture. The surface B is a finite, circular annulus on the R side of the opaque plane. The surface C is in the form of a large sphere connected to the surface B on its outer circular periphery. Note that the surface of integration C does not envelop the actual screen of observation S; otherwise, field at S would create an intractable problem in any analytical treatment. Joos and Freeman note this problem best: pp 379 in [9] from where we quote: "But in order to apply the [Kirchhoff-Fresnel] formula, the diffraction problem must, in a way, *be solved already...*" [*emphasis added*].

Kirchhoff then makes the following assumptions (**A1** through **A3**) to prescribe the constraints:

**A1** Field on the surface A is the same as that of the impinging radiation.

**A2** On the surface B, both $\tilde{U}$ and its normal derivative $\frac{\partial\tilde{U}}{\partial n}$ can be taken to be zero.

**A3** On the surface C, again, both $\tilde{U}$ and its normal derivative $\frac{\partial\tilde{U}}{\partial n}$ should be taken to be zero.

Let us consider how realistic Kirchhoff's assumptions are, in the standard view. **A1** (at the surface A) is justified if aperture diameter is much larger than wavelength, i.e., after ignoring non-uniformity and edge effects. **A2** (at the surface B) is justified if the oblique plane is electrically conducting. In the standard view, **A3** (at the surface C) is justified if the radius of the sphere is taken to be far greater than the radiation wavelength. As Born and Wolf note [5], physically this implies that Kirchhoff's formula is only valid for the times when the diffracted disturbance has not yet reached the surface C, let alone the screen S. (Really speaking, this argument from the unreached radiation is only a clever means to make the difficult surface integral vanish! Further, in a way, the argument also helps justify **A1**.)

## V. KIRCHHOFF'S ASSUMPTIONS IN THE CONTEXT OF THE HUYGENS' PRINCIPLE

**A1** implies that back-waves generated in the region R never reach surface A. Therefore, those Huygens wavelets which would have been *subsequently* generated at the surface A never quite are; their effect is eliminated from the final field solution. Such an assumption cannot be justified even if a Reciprocal Theorem is invoked. However, from a quantitative angle, of preemptive importance are the next two assumptions. **A2** and **A3** each implies that *secondary wavelets are not at all generated* at the surfaces B and C. The reason is that if the value of $\tilde{U}$ at a point is constrained to be zero at all times, then the strength of the secondary wavelets generated at that point (and of those further generated starting from these) must remain zero at all times.

## VI. THE NATURE OF OBLIQUITY FACTOR

Due to the mathematical nature of Green's Theorem (i.e., for integrals over a closed surface) the field solution at each point in the enclosed region is solely determined by the constraints placed on the surface of integration. Accordingly, the fundamental reason that obliquity factor at all comes out of Kirchhoff's analysis is because he imposes those kinds of boundary conditions.

Observe that in Kirchhoff's scheme, the radiation is required to remain finite at the aperture, but zero on the surface C. The area of C must be much greater than that of B if the surface C is to always remain beyond the reach of the farthest diffracted radiation. Irrespective of the constraints at B, since the larger surface C generates no Huygens' wavelets, almost the entire region R is denied to be affected in any way by the "back"-waves that otherwise would have propagated from C to the aperture. In contrast, Huygens' wavelets must generate at the aperture. This special asymmetry of the imposed boundary conditions is what the resultant field solution—particularly, its expression for the obliquity factor—actually captures.

Refer again to the earlier case of the unobstructed point source (Figure 2). The outer surface, at infinity, has $\tilde{U}$ as 0. Therefore, the Huygens back-waves from the outer infinite sphere cannot at all enter into analysis at any instant. Thus, there again is an imposed asymmetry, from 0 at the outer sphere to the finite value that the source produces at the inner sphere. The obliquity factor simply reflects that asymmetry. It becomes zero only in the radially backward direction but no other. In contrast, Kirchhoff does admit the finite surface A and the non-spherical surface B into analysis. This should have made the obliquity factor different from that for the point source. But, it is not [5]. The reason is, for comparative evaluation of $K_\theta$, only infinitesimal element $dS$ from A is taken; the integral is not carried out. Therefore, the obliquity factor remains identical in form to that for the point source. We can now summarize our observations thus:

*The obliquity factor encapsulates the particular geometrical relation that sources and surfaces of integration have with each other, together with the imposed initial value- and boundary conditions. It is not a fundamental feature of the differential wave equation or of diffraction phenomena.*

## VII. DIFFRACTION THEORY SOLUTIONS; ISOTROPY OF THE HUYGENS WAVELETS

The standard view attributes anisotropy to Huygens wavelets because $K_\theta$ carries it. But right since Fresnel's day, Huygens' Principle has not been correctly applied. Fresnel considers only the first set of secondary wavelets: those that originate from the imaginary sphere over which his zones are constructed. The proof [5] does not consider the next set(s) of wavelets for every integral $\lambda$ progress of the wave-front. Yet, as per Huygens Principle, the fundamental building blocks of the process—the wavelets—act only locally, and the non-uniformity of the final field solution evolves according to the manner in which the *local* cascades and feedbacks progress through the domain. Thus, Fresnel assumes secondary waves, not wavelets. The quantitative result is correct only because the proof considers semi-infinite space after diffraction. As to Kirchhoff's theory, there is no mechanism to accommodate cascades and feedbacks in Green's theorem, only their final effect. To conclude, the analytical diffraction solutions cannot be directly mapped on to the building blocks of the Huygens process, and *vice versa.*

Obliquity factor thus essentially boils down to a *globally* applicable measure of the intensities with which local cascades and feedbacks occur in different directions in a *particular* solution; such intensities being averaged out over the entire domain and time duration; the measure being applicable only to that particular solution—not in general.

On the other hand, the question of whether Huygens wavelets are isotropic or not, is a local issue, determined primarily by the nature of the governing differential field equation, and therefore having reference to the local infinitesimal neighborhood. The global parameters of domain geometry and boundary constraints are in principle to be omitted in settling such a question. Consequently, it can only be resolved in reference to the nature of the *fundamental* solution of the differential equation. Owing to the symmetry in the wave equation (and assuming homogeneous isotropic material) the Huygens wavelets turn out to be necessarily isotropic. As a consequence, not just back-waves but intensities *at any angle whatsoever* must be kept unmodified. Even if locally isotropic, once the constraints are imposed, the infinitely many local Huygens wavelets will together produce the non-uniform field solution pattern through cascading and feedback.

## VIII. A BRIEF NOTE ABOUT A CASE THAT *SEEMS* COUNTER TO THE PRESENT VIEW

For lack of space, we could not include a discussion of waves on pond surface due to a pebble, though the present view is consistent with the phenomenon. In essence, our argument touches on: (i) the distinction between pulse-like disturbance on the surface of the pond and the wave PDE-governed processes occurring within the domain volume; (ii) Green's functions and the sharpness of the signal in spaces of different dimensions; (iii) taking the pebble as a moving source, and the radial symmetry in the horizontal surface due to the locus of its motion; etc.

## IX. MAIN PRACTICAL COMPUTATIONAL BENEFITS OF THE NEW VIEW

First, consider the disadvantages of applying obliquity factor in practical numerical analysis:

1. Computational algorithms using obliquity factor cannot be made geometry-independent.

2. Anisotropic obliquity factor implies having to specify or compute the direction in which the radiation source lies for every point of space within the domain.

3. Anisotropy implies additional computations for all the directions in which propagation occurs.

On the other hand, our view provides a geometry-independent, general, far simpler, and computationally cost-effective method. For an interesting application, see [10, 11].

## X. CONCLUSIONS

1. The issue of obliquity factor has been analyzed. The specific mathematical step in Kirchhoff's theory from where the term arises has been identified. The physical assumptions behind the step and their implications from viewpoint of Huygens-Fresnel Principle have been discussed.

2. The obliquity factor is shown to capture the geometrical relation of sources to the surfaces of integration and the initial- and boundary-conditions. Thus, it is a global "measure" of particular solutions. It is fundamentally inapplicable to characterize the local character of a differential equation field. The discussion is motivated by an appeal to the nature of the Huygens process as a feedback-based cascading process.

3. A new view having the following attributes has been put forth:

   (a) The new view denies the idea of obliquity a fundamental role in analysis of waves, diffraction, or Huygens-Fresnel Principle.

   (b) The new view treats (for a homogeneous and isotropic medium) both back-waves and front-waves, indeed Huygens waves at any angle whatsoever, at par with each other, i.e., as indirect processes that may not be directly observed as physically separate waves but which together give rise to the actually observed wave patterns.

   (c) Huygens wavelets are taken to be isotropic following the symmetry argument.


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