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Obliquity factor is not essential to the Huygens-Fresnel principle

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The men…

Huygens
1629 - 1695

Fresnel
1788 - 1827

Helmholtz
1821 - 1894

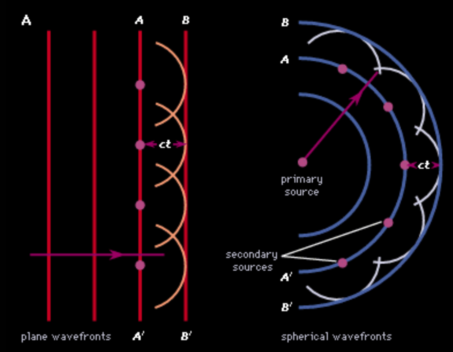
Kirchhoff
1824 - 1887

Outline

- The standard view of obliquity factor
  - Meaning, and the arguments for its necessity from Huygens' Principle
  - How Fresnel used (and unwittingly modified) Huygens' Principle
  - Helmholtz-Kirchhoff integral theorem
  - Kirchhoff's diffraction formula
- The Huygens process, reconsidered
  - Our view, as contrasted from Fresnel's (and Huygens'!)
  - Kirchhoff's theory and Huygens' wavelets
- What does our view really mean and imply?
  - Obliquity Factor
  - Huygens' Process: cascading, feedback-based
  - Theoretical and computational benefits

Huygens' Principle
(1678)

- Explains how waves propagate
- Each point on the old wave-front acts as a new "source"
- Wavelets are spherical <u>surfaces</u> of radius λ
- <u>Envelope</u> of new wavelets gives the new wave-front
- The process <u>repeats</u>
- Back-waves not at all discussed
- Explains why waves curve around obstacles

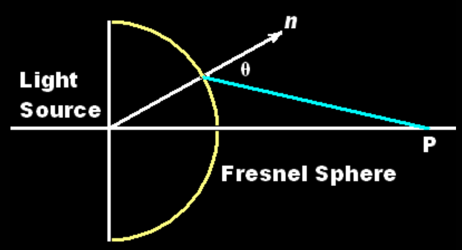


- Figure Credit: taken from Encyclopedia Britannica, Year 2000 CD Edition

Fresnel

The first theory of diffraction (1818)

- Constructs Fresnel zones on a single sphere, and adds contribution of each zone at a distant point
- Considers only the first set of Huygens' wavelets—those emitted from the sphere. So, these really are waves!
- Secondary waves from each zone mutually interfere
- Fresnel introduced interference in Huygens' Principle



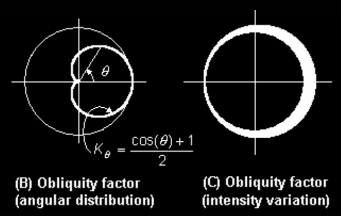
Obliquity Factor, the Idea

- Only Huygens' Principle can show why waves curve around obstacles
- In reality, waves seem to *expand*. They seem not to go back towards the center
- But secondary waves of Huygens' Principle would travel back towards the center
- To keep the Principle, modify the wavelet!
- So, progressively reduce amplitudes in "backward" direction

Obliquity Factor

Precisely what form of anisotropy is necessary?

- 1818: Fresnel *assumed* semi-spherical obliquity
- 1849: Stokes calculated the (1 + cos θ)/2 distribution
- 1872: Kirchhoff's mathematically rigorous analysis. Used wave PDE. Gave the same result.
- 20th Century: The obliquity idea (and the distribution) remained unquestioned



Kirchhoff's Analysis

The main steps of his diffraction theory (1872)

- Consider wave PDE in complex amplitude
- Solve Helmholtz Equation using Green's Theorem
- Result: Helmholtz-Kirchhoff Integral Theorem (HKIT)

- Substitute the complex space-dependent part of the wave into the integral theorem (HKIT)
- Result: Fresnel-Kirchhoff Diffraction Formula (FKDF)

- Consider suitable diffraction geometry
- Apply "reasonable" boundary conditions
- Evaluate the integral of the diffraction formula (FKDF)
- Result: Contains an evaluation of obliquity factor

Mathematical Background
for the Helmholtz-Kirchhoff Integral Theorem

- Scalar wave equation (complex amplitude)

$$\nabla^2 \tilde{\phi} = \frac{1}{c^2} \frac{\partial^2 \tilde{\phi}}{\partial t^2}$$

- Consider 3D, spherical, expanding wave. Split it into unspecified space- and time-dependent parts

$$\tilde{\phi} = \tilde{U}\tilde{T} = \left(\frac{\tilde{\phi}_0}{R} e^{ikR} \right) \left(e^{-ikct} \right) \qquad \tilde{\phi} = \tilde{U}_{unspecified} \, e^{-ikct}$$

- Helmholtz Equation governs the space-dependent part

$$\nabla^2 \tilde{U} + k^2 \tilde{U} = 0$$

Green's Theorem

- Divergence Theorem of Gauss

$$\iiint_T \vec{\nabla} \cdot \vec{F}\, dV = \oiint_S \vec{F} \cdot \vec{n}\, dA$$

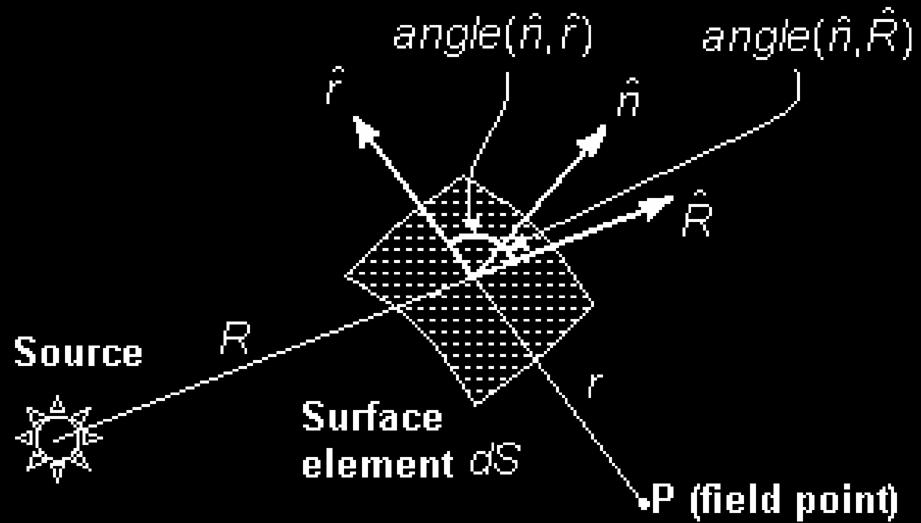
- Green's Theorem

$$\vec{F} = f\,\vec{\nabla}\, g$$

$$\iiint_T \left(f\,\nabla^2 g - g\,\nabla^2 f \right) dV = \oiint_S \left(f\,\vec{\nabla} g - g\,\vec{\nabla} f \right) \cdot d\vec{S}$$

Helmholtz-Kirchhoff Integral Theorem

- Solve Helmholtz Equation using
  Green's Theorem
  - Choose f and g as…



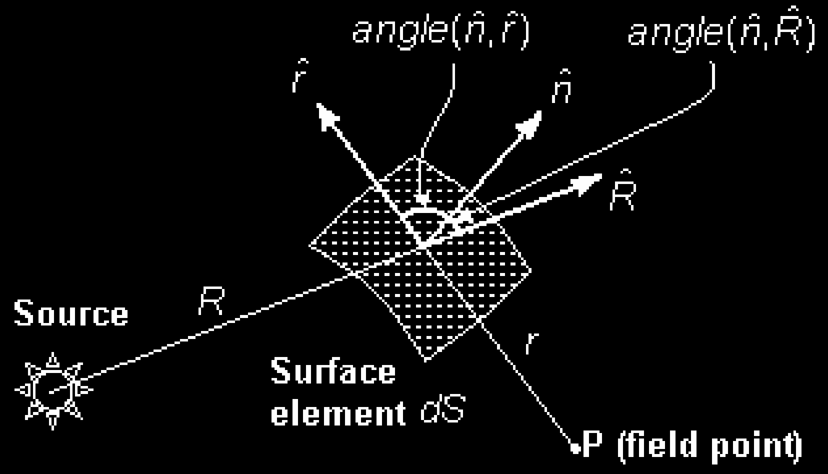
$$f = \tilde{U} \qquad g = \frac{e^{ikr}}{r}$$

$$\tilde{U}_P = \frac{1}{4\pi}\left[\oint\!\!\oint_S \left(\frac{e^{ikr}}{r}\right)\vec{\nabla}\tilde{U}\cdot d\vec{S} - \oint\!\!\oint_S \tilde{U}\,\vec{\nabla}\left(\frac{e^{ikr}}{r}\right)\cdot d\vec{S}\right]$$

Fresnel-Kirchhoff Diffraction Formula

- Substitute U into H-K Integral Theorem and get:

$$\tilde{U}_P = -\frac{i\tilde{U}_0}{\lambda} \oiint_S \frac{e^{ik(r+R)}}{rR}\left[\frac{\cos(\hat{n},\hat{r}) - \cos(\hat{n},\hat{R})}{2}\right] dS$$



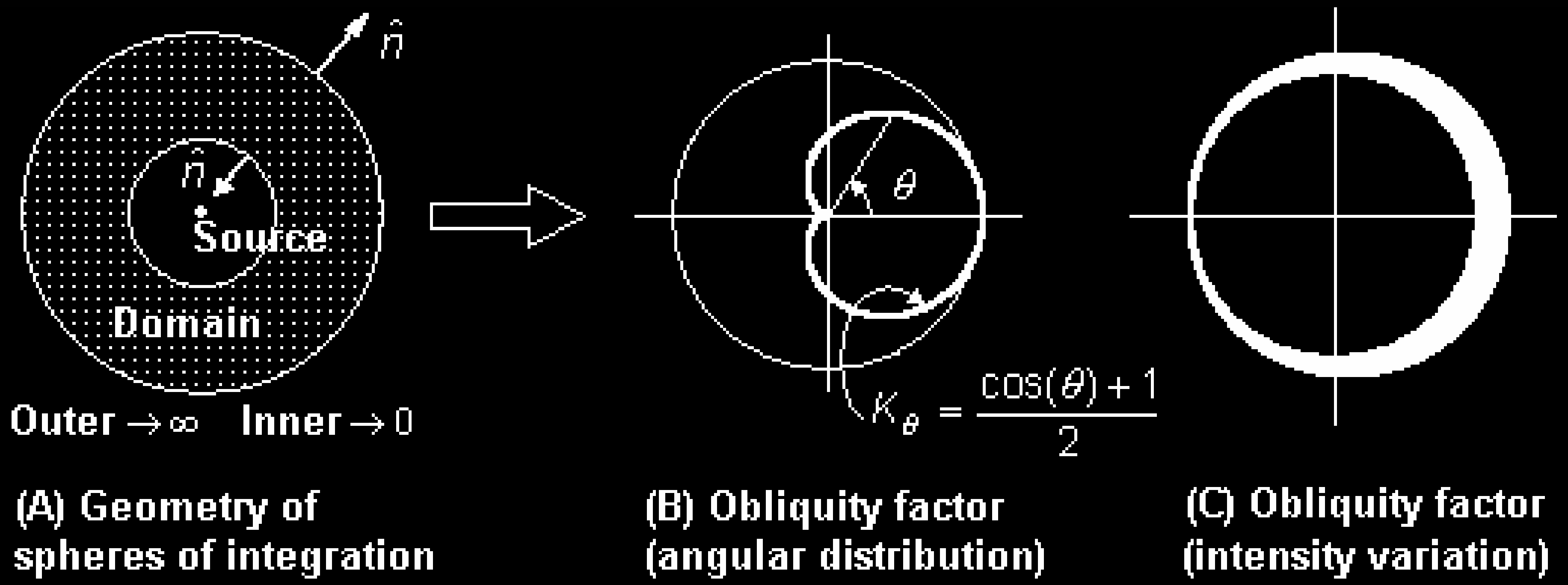
- Obliquity Factor: Generic Form

$$K_\theta = \frac{\cos(\hat{n},\hat{r}) - \cos(\hat{n},\hat{R})}{2}$$

Fresnel-Kirchhoff Diffraction Formula

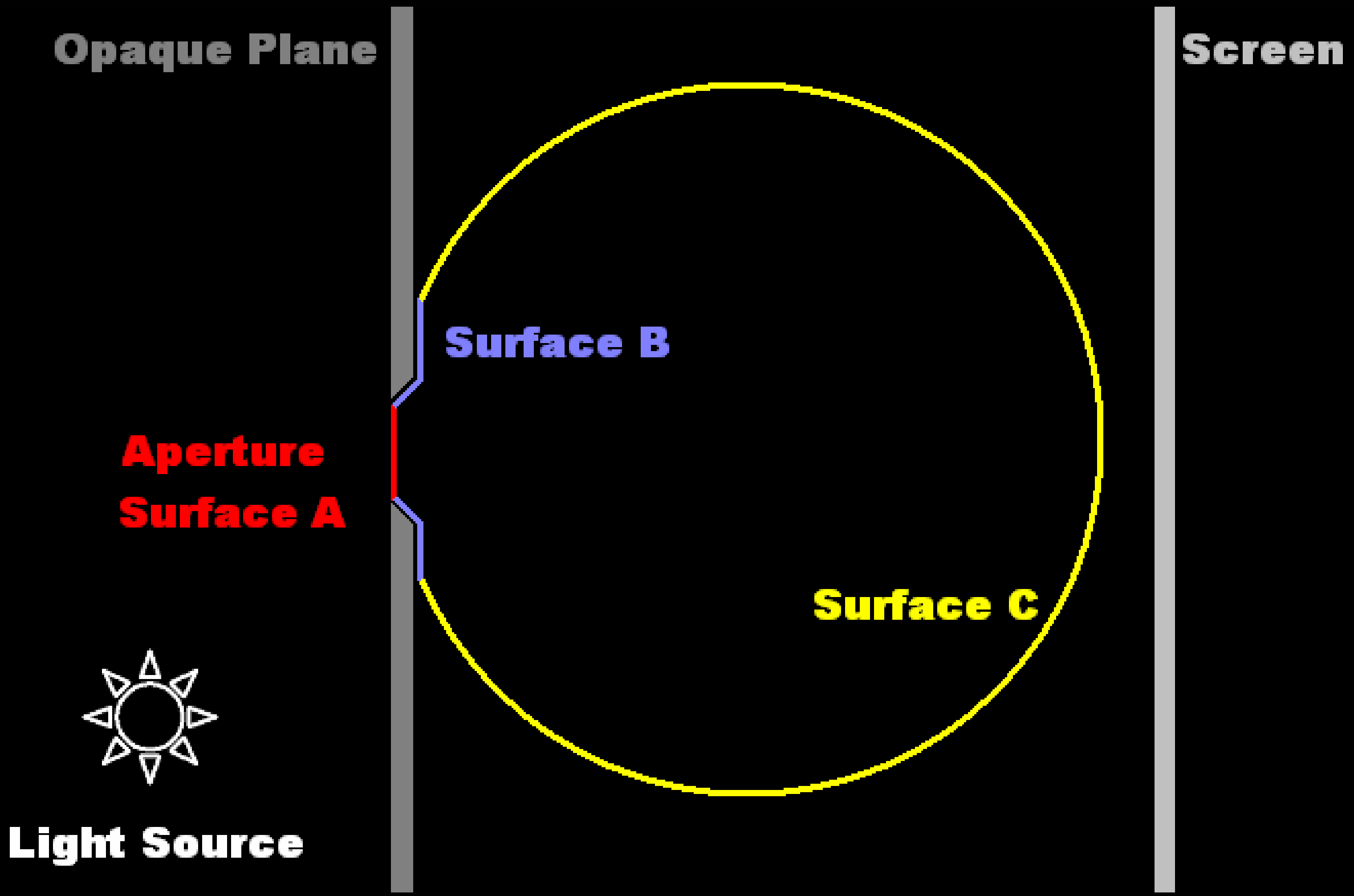
A simple example of how to evaluate it for a geometry

- Point source in infinite space



Kirchhoff's Analysis

The geometry of aperture diffraction



Kirchhoff's Assumptions

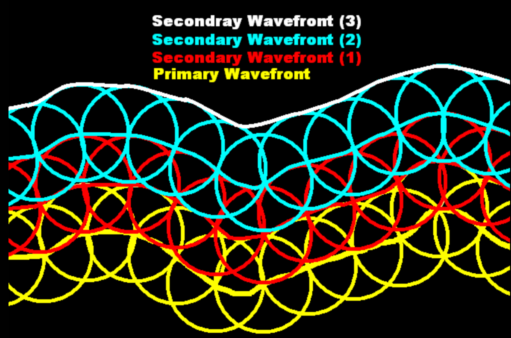
Justifications in the standard view

- A1: Field at aperture is the same as the impinging radiation
- A2: Both U and dU/dn are zero at surface B (i.e. annular ring)
- A3: Both U and dU/dn are zero at surface C (i.e. the balloon)

- Non-uniform field, Edge effect are ignorable
- The opaque plane is electrically conducting
- radius » λ
  - "Unreached Radiation"

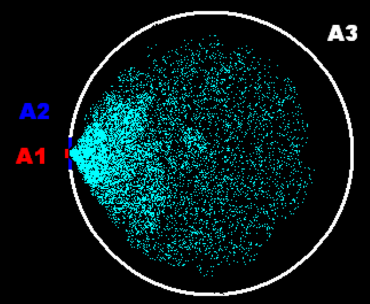
Huygens' Principle, Our View

- Wavelets, not waves
  - Contrast to Fresnel
- Wavelets interfere
  - Contrast to Huygens
- Isotropic spread
  - Contrast to *all* previous views and theories
- Huygens Process
  - We introduce the term!
  - Spatially, cascading
  - Temporally, feedback-based



Kirchhoff's Assumptions:
Objections in our view

- A1: Huygens' back-waves never *reach* aperture.
  Is this justified…
  - by Huygens' Principle? No!
  - by reciprocity Theorem? No!
- A2: Secondary waves (back, <u>or front</u>) are *never generated* at the annular cap B
- A3: Again, no generation of the secondary waves, at the very large surface C!



Kirchhoff's Boundary Conditions and Huygens Wavelets

- At aperture, Kirchhoff's boundary conditions…
  - Allow generation of Huygens wavelets
  - Deny Huygens' back-waves to reach aperture
- At surfaces of integration, Kirchhoff's boundary conditions…
  - Deny generation of any Huygens' wavelets
  - Deny aperture-generated Huygens' forward waves to reach the spherical surface
- We conclude
  - Obliquity Factor only captures this special asymmetry imposed via the particular boundary conditions

Further…

- With Kirchhoff's boundary conditions, no Huygens Process can at all connect aperture with the other surfaces of integration

- Fresnel-Stokes Theory considers only the first set of wavelets
  - Still successful because only infinite domains considered

- Effectively, Kirchhoff's Obliquity Factor equals Fresnel-Stokes' Obliquity Factor, only if aperture size << sphere of integration
  - Non-uniformity could have been included in Kirchhoff Theory

Our View:
The nature of obliquity factor

- Obliquity Factor encapsulates the particular geometrical relation of sources/sinks with integration surfaces
- Obliquity Factor is <u>not</u> a fundamental feature of
  - Diffraction
  - Wave Equation
- Obliquity Factor is a global measure (“averaged” over the entire domain) of how the field is distributed

Our View:
The “shape” of Huygens’ wavelets

- What “shape” is a local issue
  - determined by what is valid at each point, separately
- Therefore, determined by the nature of the differential equation
  - not integration over a specific region or geometry
- Wave PDE has symmetry in space axes (and also time)
- Therefore, Huygens’ wavelets may be taken to be spherical
  - But note, time dimension is complex!
- The globally un-isotropic solution will automatically evolve through spatial cascades and temporally separated feedbacks

Our View:
Back-waves and pond waves

- Secondary wavelets in all directions are necessary to produce the physically observed field pattern
- Front-waves and Back-waves are *indirect* processes, not directly observed as physically separate waves
- Pond waves are not 3D PDE-waves, in the true sense. They are pulse-like result of the indirect processes within the water volume
- Analysis of pond waves would involve issues like:
  - Dimensionality of spaces
  - Greens' functions and sharpness of signal
  - Pebble as a moving source

Our View:
Computational benefits

- Geometry Independence
  - Algorithms can handle arbitrary geometry

- For calculations at each point in domain…
  - "Domain points need not know where the sources lie!"
  - No need to separately compute the direction in which each source lies

- For calculations at any given point in the domain…
  - No separate calculation in different directions

Contrast to Earlier Views…

- Our view seems to be in contrast to *all*…
  - Huygens' writings (1678, pub.1690)
  - Fresnel's view (1818)
  - Stokes' derivation of Fresnel diffraction (1848)
  - Kirchhoff's diffraction theory (1872)
  - All modern textbooks
    - Born and Wolf (1999), Lipson (2001), Hecht (2003)

- …But our view has <u>no mathematical contradiction</u> to existing theories
  - Infinite domains-based theories in physics, mathematics

- Our view of the Huygens <u>Process</u> (a new term!) …
  - Has cascades and feedbacks (new characterization)
  - Is in harmony with what the original Huygens' Principle suggests

An Interesting Application...

- Resolution of the quantum wave-particle duality

Thank You!