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A New Numerical Approach for Modeling the Ideal Fluid Flow on Computer

A. R. Jadhav
Student of Ph. D. in Mechanical Engineering
(Formal registration pending)

P. P. Chikate
Doctoral Research Guide in Mechanical Engineering

University of Pune, India

A New Numerical Approach for Modeling the Ideal Fluid Flow on Computer

Outline

- **Equation of the Ideal Fluid Flow**
- **A New Approach to Solve Helmholtzian Fields**
  - Conceptual Outline and a Fundamental Theorem
  - Differences from the Nearest Prior Work
- **Computer Trials and Results**
- **Main Features of the New Approach**
  - Similarities to and Differences from FEM and/or FDM
  - Computational Costs
- **Conclusions**

Equation of the Ideal Fluid Flow

The Equation of Ideal Fluid Flow

$$\nabla^2 \phi = k, \quad \vec{v} = \vec{\nabla}\phi$$

- Kinematical equation (description of motion)
  - Surface and body forces not considered in the analysis
- Linear analysis of continuity
- Singularities (sources and sinks) included
- It is Helmholtzian in form

$$\nabla^2 \phi = f(\phi) \qquad f(\phi) = \frac{\partial^2 \phi}{\partial t^2} \text{ or } \frac{\partial \phi}{\partial t} \text{ or } k$$

The Assumptions: How good are they?

- Irrotational
  - Absence of element rotation implies inviscid flow
  - The flow of “dry” water – John von Neumann
  - Boundary layer is absent in the analysis (no eddies, no drag force)
- Steady-State
  - Not suited to address the cavity filling problems
  - No treatment of transients / propagation of shocks
- Incompressible
  - OK at low Mach numbers, esp. for liquids

Announcing a New Approach
to Solve
Engineering Field Problems

Towards Formulating the New Approach

- Key Observation:
  - The equation of Ideal Fluid Flow (Potential Flow) is Helmholtzian in form
  - The equation of Wave Propagation is Helmholtzian in form

- Apply a technique of Geometric Probability
  - To the phenomenon of wave propagation

- Exploit the similarity
  - From: Wave Equation
  - To: Potential Flow (i.e. Ideal Fluid Flow)

About Geometric Probability

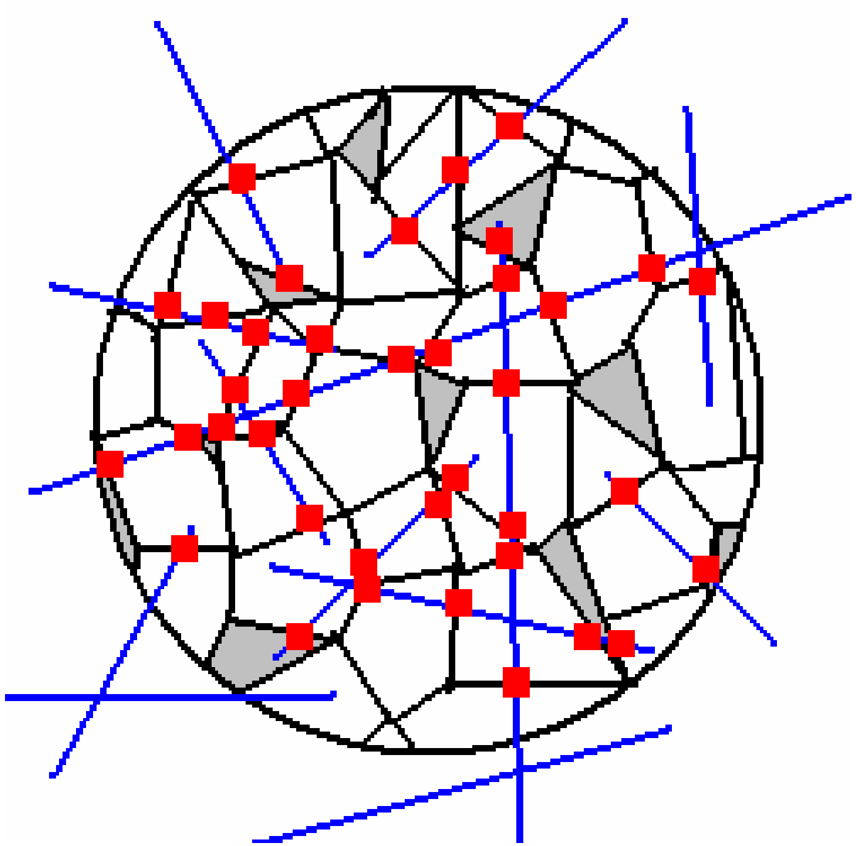
- What Is Geometric Probability?
  - Used in <u>quantitative</u> metallurgical microscopy
  - Applies probability theory and calculus to estimate geometrical features of 3-D objects from their 2-D microstructural sections

- What is the main idea in geometric probability?
  - Geometrically sample the feature of interest
    - Example (Points Probe): Point fraction => Volume fraction
    - Lines or curves also can be used as probes for sampling
  - For advanced problems, use calculus for integration in 3-D
    - Implicit assumptions on randomness and integration scheme can both be misleading!!

An Example of a Geometric Probability Technique

Problem: Estimate the surface area of grain boundaries per unit volume.

Solution: 1. Place line probes at random
2. Count number of intersections per total probe length



$$S_V = 2\overline{P}_L$$

$$L_A = \frac{\pi}{2}\overline{P}_L \qquad L_V = 2\overline{P}_A$$

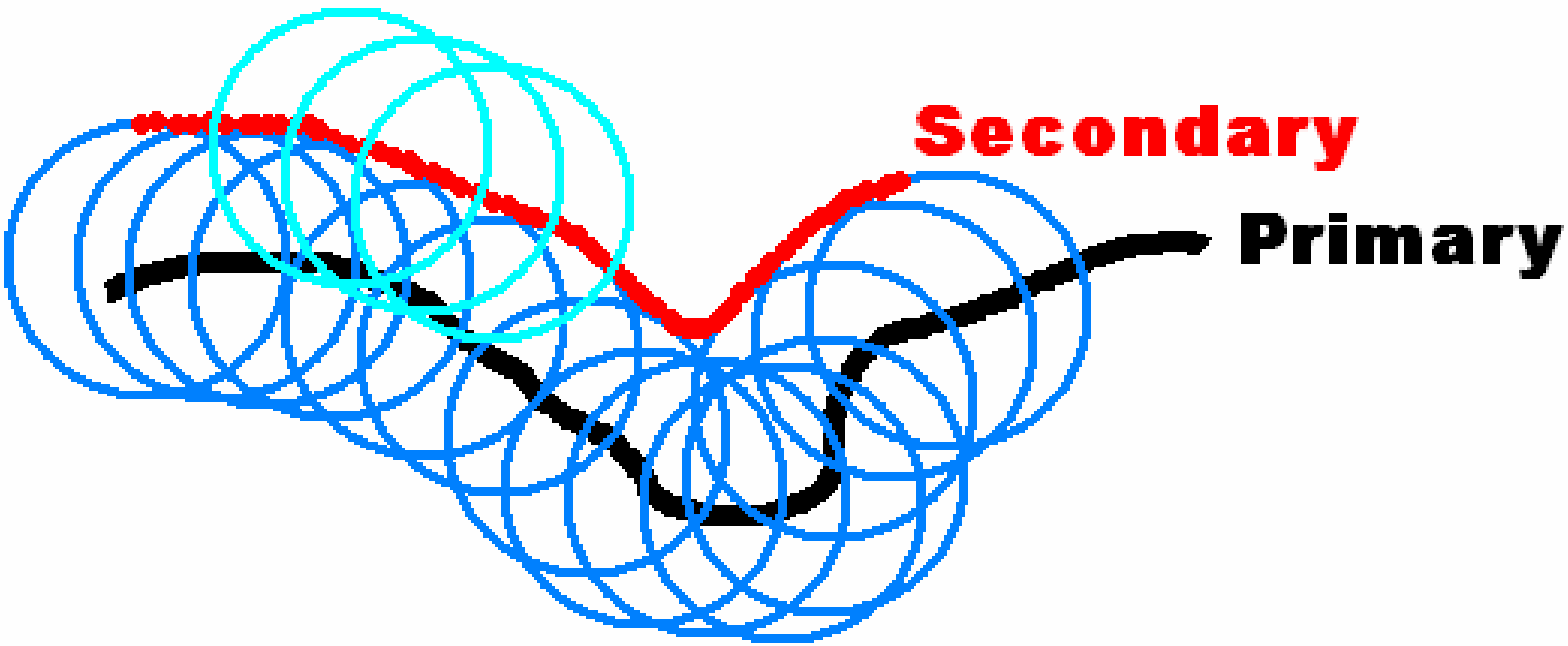
Conceptualizing the New Approach

- We will apply a technique from geometric probability to the problem of wave propagation

- Then, we will take these observations to the context of ideal fluid flow

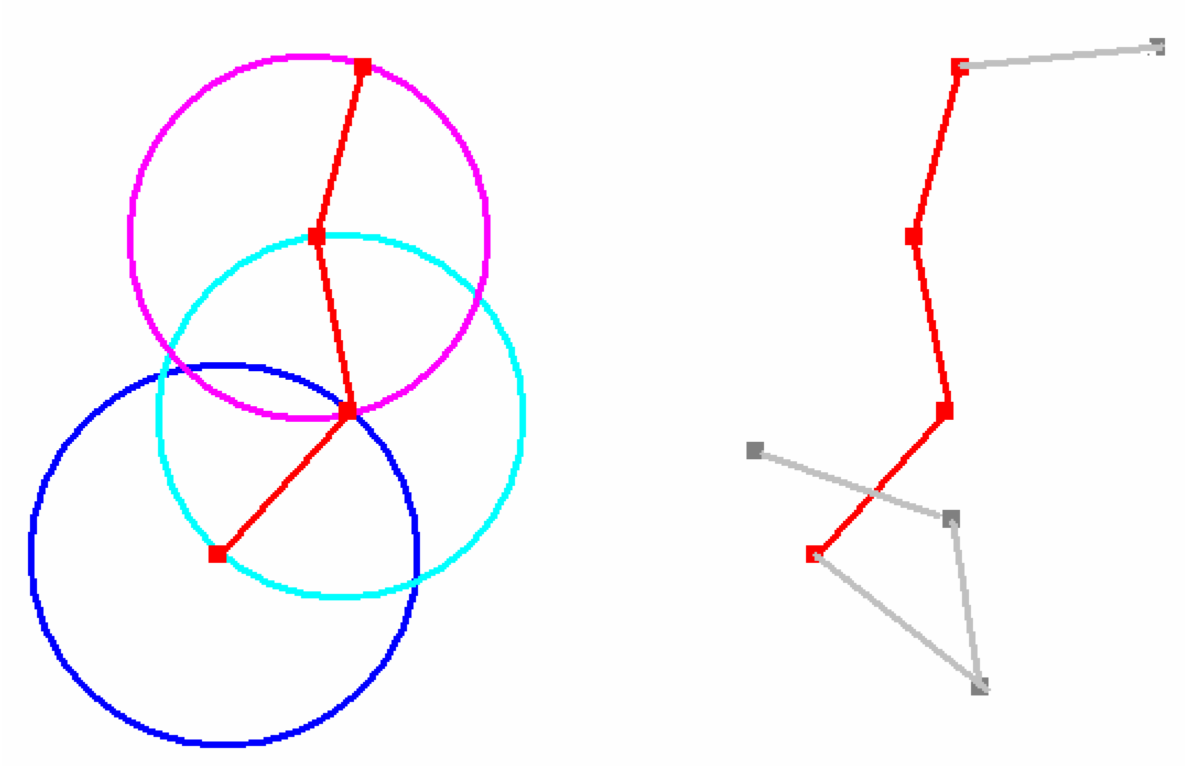
- We assert, the approach can be extended to any Helmholtzian field

How Waves Propagate: Huygen's Principle (1690 A.D.)



The Basic Idea: Apply geometric sampling to Huygen's waves

Sample each successive secondary wave using a line probe. Join these displacement vectors in order.



Essentials of the New Approach

- Basic Idea
  - Geometric sampling of Huygen's waves
- Huygen's Principle
  - For finite size of secondary waves (integral λ)
    - Sampling yields: Random Walk "Motion"!
  - Wave Solution is approached when:
    - Secondary sources are <u>infinitesimally close</u>
- Solution for Continuum Field in Finite Objects is obtained when:
  - Secondary wave is <u>infinitesimally small</u> in size
    - Adjust Huygen's principle mathematically for phase
  - An <u>infinite number of particles</u> are involved
    - New! Important for engineering applications!!

A Theorem Fundamental to the New Approach

The superposition of
an infinite number of particles
each randomly sampling its succession of
infinitesimally small
Huygen's (secondary) waves
yields a complete description of wave field.

Then, by implication, <u>any</u> Helmholtzian field!

The Fundamental Theorem Restated (in random-walk terms)

The superposition of
an infinite number of particles
each undergoing
infinitesimally small displacements
in random-walk
is a solution
to any Helmholtzian field problem.

Either of these equivalent statements can be taken as implying a new interpretation for the Laplacian operator (∇^2) itself !!

Applications

- **Nearest Prior Work:**
  Random-Walk in Fractals and Chaos Theory ($1 < d < 2$)
  - Diffusion-Limited Aggregation (DLA)
  - Fractal Fluid Motion
  - Neuronal Growth, etc.

- **Present Work:**
  Classical Fields in Engineering ($d = 2$)
  - Fluid Flow
  - Groundwater Seepage
  - Torsion
  - Heat Conduction
  - Diffusion
  - Acoustics
  - Waves
  - Electromagnetic Fields, etc.

Computer Trials and Results

Putting It to Test…

- Two Case-Studies in 2-D Flows
  - Infinite Planar Flow
    - From circle at infinity towards a sink in the center
    - Benchmark: Closed-Form Analytic Solution
  - Finite Planar Flow
    - Flow past a square obstacle in a square cavity
    - Benchmark: FEM Results

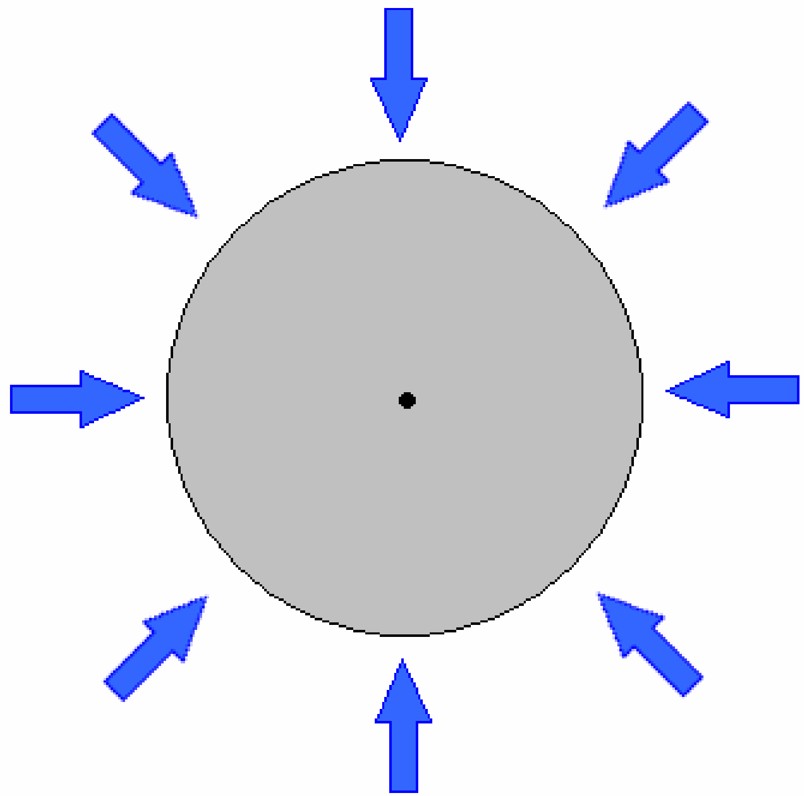
- Implementation on Computer
  - Computing Environment
    - A desktop PC: Intel Pentium III @ 933 MHz; 512 MB RAM
    - Microsoft Visual C++ 6.0 on Windows 2000 Professional
  - Custom-written code
    - for both FEM and the New Approach

Case 1: Infinite Planar Flow Benchmark: Analytic Solution

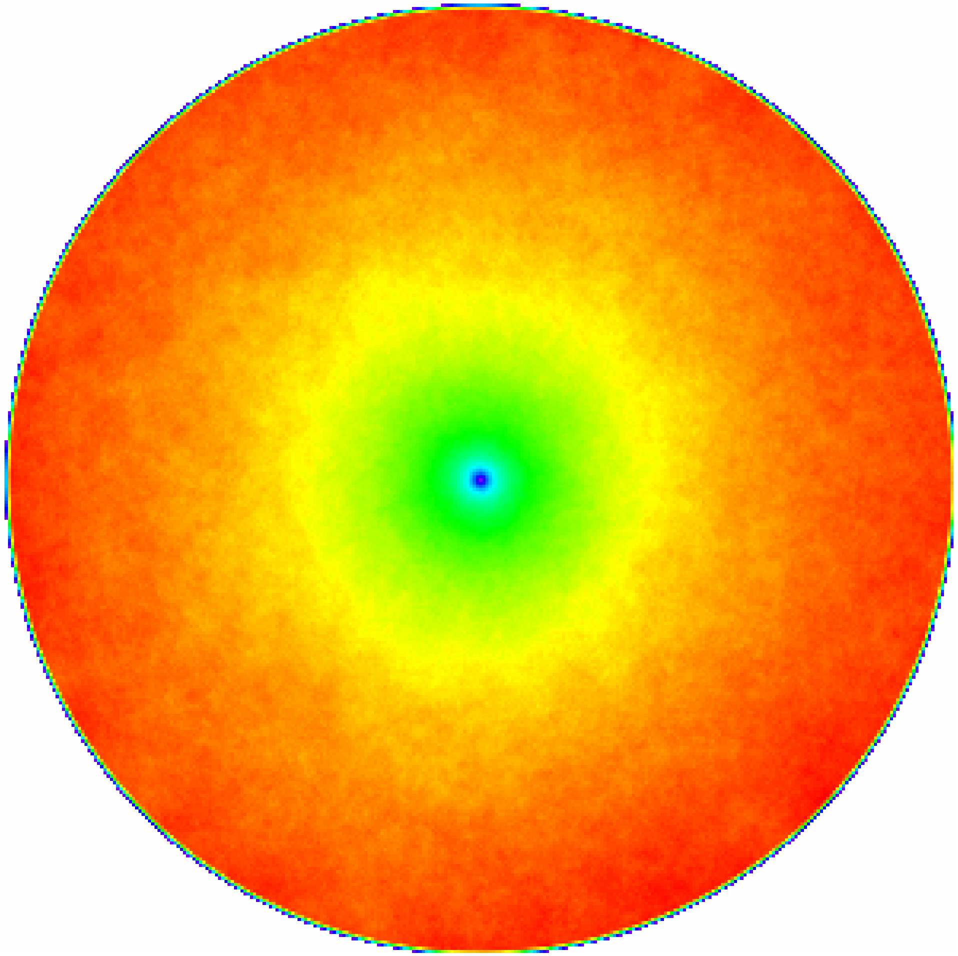
Flow from
a circle at infinity
towards
a singular sink
in the center.

Velocity-Potential
is given as:

$$\phi = \frac{q}{2\pi} \ln(r)$$



Result for Infinite Planar Flow: Velocity-Potential Field

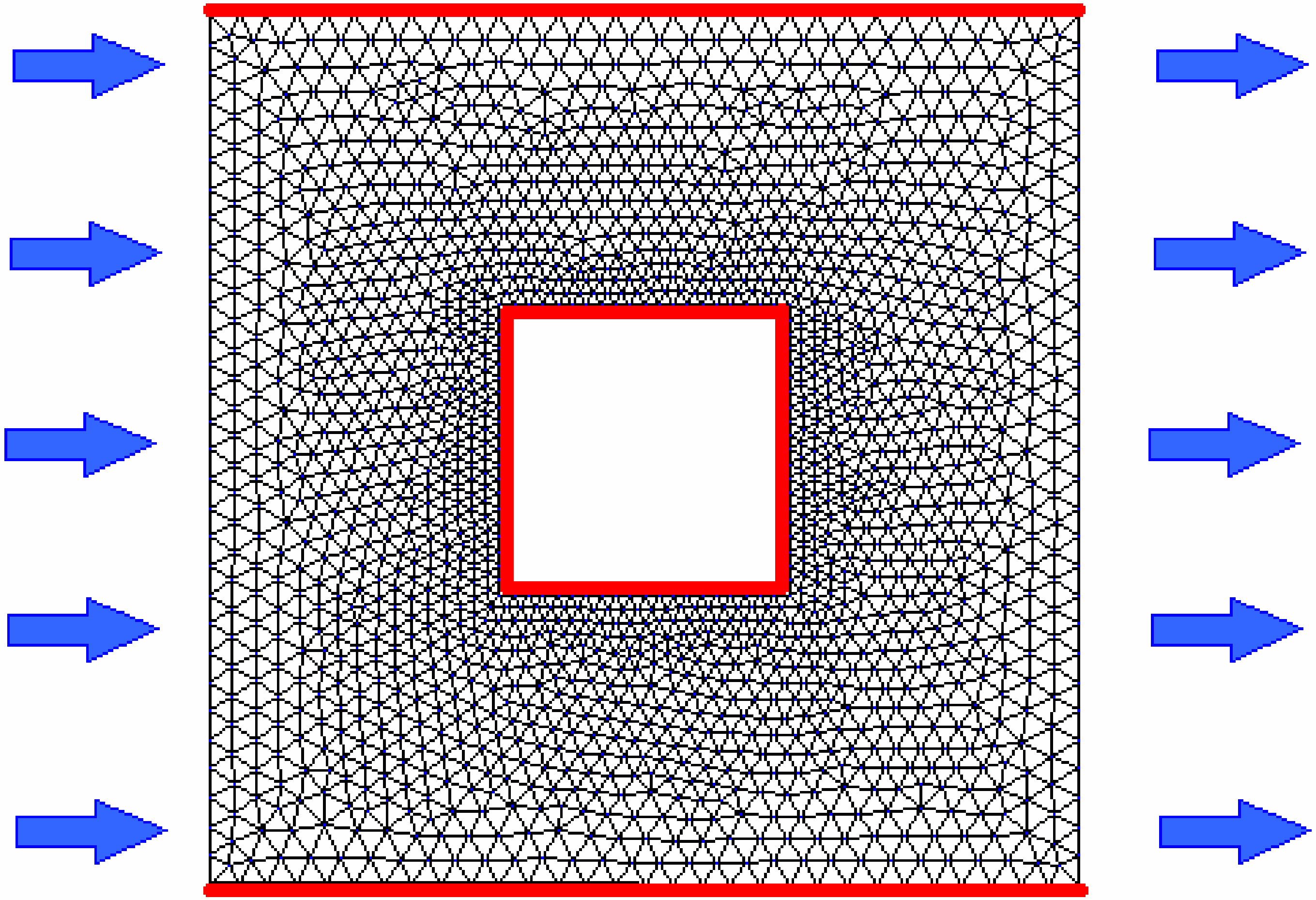


Case 2: Finite Planar Flow Benchmark: FEM

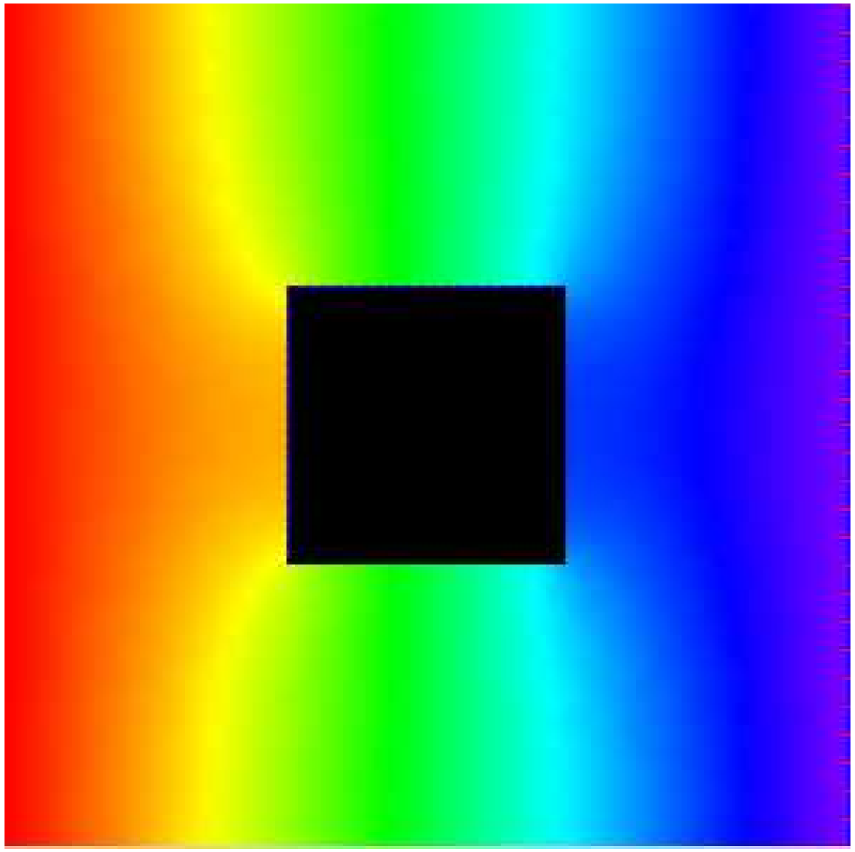
FEM Mesh

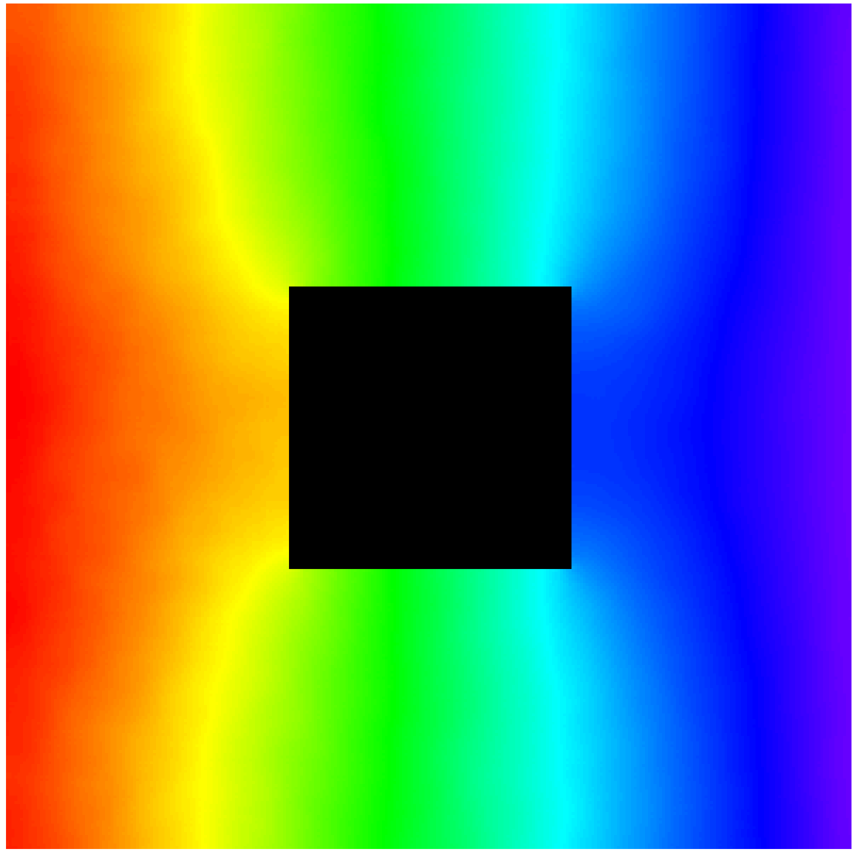
Nodes: 1,693

Elements: 3,186

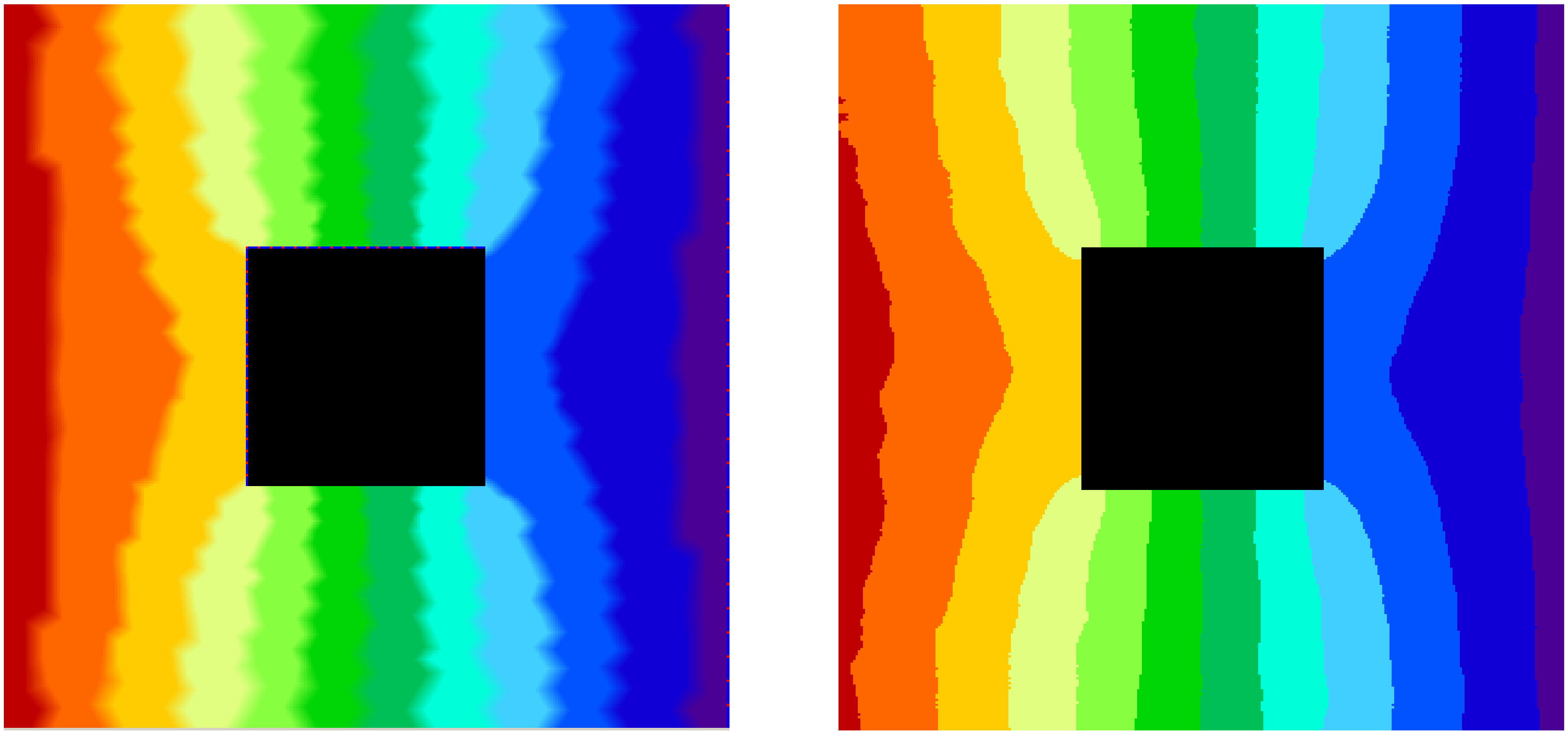


Results of FEM and New Approach: Velocity-Potential Field





FEM and New Approach: Discretization Effects



Comparisons of Errors: Within 5% (for smoothened data)

- Comparisons are given for velocity-potentials
- Compared to Closed-Form Analytic Solution
  - Time for analysis: 30 minutes
  - Errors: Within **3.5%**
- Compared to FEM
  - Analysis time same as that for FEM: 1 min 30 sec
  - Deviation from FEM results is within **5.0%** for smoothened data, except near the edges
  - Analysis with the new approach improves with longer analysis times

Errors: Some Clarifications

- Why do the errors arise?
  - Number of particles cannot be infinitely large
  - Displacement cannot be infinitesimally small
- What is the nature of the errors?
  - Statistically random scatter
- How do you reduce the errors?
  - Longer analysis time (parallel processing)
  - Still under study…
    - Better handling of geometry and boundary conditions
  - Brute-Force: Smoothen out ! (Not too bad)

Main Features of
the New Approach

Main Features Common to FEM and/or FDM

- The new approach can handle
  - Finite objects
  - Arbitrary geometry
  - Inhomogeneity
- It is necessary to
  - Discretize the domain (space & time)
  - Suitably adapt boundary conditions
  - Interpolate the solutions
  - Validate and interpret the results

Main Features Different from FEM and/or FDM

- No simultaneous (or matrix) equations
- No meshing or refinement (saves effort!)
- Geometrical representation of objects differs
- Singularity handling is straight-forward
- Analyst can be kept updated during analysis

Some Features as a Computational Method

- Where are the computational costs?
  - Time-wise: In the refinement of results
  - Space-wise: Probably, a data intensive method

- Computational power requirements
  - Within an order of magnitude of FEM/FDM
  - A desktop PC is enough!
  - Moore’s Law is definitely encouraging

- Inherent ease of parallel processing
  - Few data are exchanged during processing
    => Simplest model (MIMD) of parallel processing
  - Therefore, farm of inexpensive smart nodes (SETI!)

Where do I apply the New Approach (in fluid mechanics)?

- First, it addresses the Potential Flow
  - Ideal fluid flow mainly serves as a base-line analysis for more complex flows
  - Yet, a direct practical example: flow around airfoils

- Then, some viscous flows too…
  - For steady-state incompressible flow through duct, the Navier-Stokes Equation reduces to:

$$\nabla^2 u = \frac{1}{\mu}\frac{\partial p}{\partial x}$$

Physically, here, u is the velocity component along the duct length; it is not the velocity-potential.

Conclusions

Conclusions

- **This is the very first paper!**
  - For the analysis of an engineering (i.e. classical) field problem
- **A fundamental theorem is stated**
- **The method is FAST enough!**
  - No modification whatsoever required for parallel processing
  - Yet, a single desktop PC can easily handle the method !
- **The method is ACCURATE enough!**
  - Infinite domain: Closed-form analytic solutions
  - Finite domain: FEM results
- **Main features are discussed**
  - And compared to FEM and FDM
  - Promises to be a new tool of design
- **Lot of work still needs to be done!** (Ph. D.!!)

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- Software for FEM mesh generation
  - “EasyMesh” by Niceno
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Thank You!